The Crisis of Expertise*

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Abstract

A careerist expert sells advice to decision makers who doubt that the expert is truly informed. I find that a "crisis of expertise" can emerge, in which a decision maker dismisses an informed expert's correct advice and relies only on public information to guide his action. Remarkably, this crisis happens if and only if public information has mediocre quality, and so high-quality public information helps the decision maker efficiently utilize the informed expert's knowledge. My analysis elucidates a novel complementarity between the quality of public information and the quality of expert advice.

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1 Introduction

The Internet has two faces. It provides an abundance of public information for decision makers to utilize. In finance, for instance, a study by TIAA (2021) finds that investors increasingly rely on social media contents rather than on professional advisors. The Internet also exposes decision makers to fake experts. Take, for example, the case of Patricia Russell, who appeared in 2019 on *LinkedIn* as a certified financial planner; Russell's financial advice was quoted in major outlets, allowing her to benefit from an undeserving reputation as a financial expert.[1] These two faces of the Internet are one typical explanation for decision makers' increasing reliance on public information online rather than on expert advice, including genuine advice from informed experts (see, e.g., Nichols, 2017).

The purpose of this paper is to understand how decision makers value expert advice given the two faces of the Internet, thereby addressing the following important questions. How does the quality of public information affect decision makers' reliance on expert advice? Is decision makers' declining reliance on expert advice merely a phenomenon in which they substitute high-quality public information for expert advice? Finally, as decision makers increasingly utilize public information, countries worldwide adopt efforts to safeguard the quality of public information (see, e.g., Funke and Flamini, 2021); how do these efforts affect social welfare?

I study a model in which a career-concerned expert sells advice about a randomly evolving state over two periods. In each period, a new principal enters and must take an action, such as an investment decision. He hires the expert because his desirable action depends on the state but he only has access to noisy public information about this state. Only the expert knows whether she is an informed type who sees the state or an uninformed type who, like the principals, does not see the state. This expert faces career concern in the first period because her performance, consisting of her message and what the state turns out to be, is observable to the future principal and affects her future wage. I study reputation equilibria in which both expert types benefit from a

[1] See, e.g., "Fake 'expert' diminishes the value of genuine financial help," *The Seattle Times*, August 24, 2019.

higher reputation, namely a higher future principal's belief that she is informed.

1.1 The crisis of expertise

My goal is to understand when and why a principal matches his action with the state that public information deems most likely to be true, irrespective of the informative advice that he receives from the informed expert—I call this phenomenon a "crisis of expertise." This crisis does not happen in the second period. The reason is that in reputation equilibria, the expert collects her reputation benefit through a positive second-period wage and a principal pays this wage only if the expert's advice is influential in his decision-making.

My main result characterizes the crisis of expertise in the first period and shows that under natural conditions, this crisis happens if and only if the quality of public information about the state is neither too high nor too low, just mediocre. Moreover, as it turns out, the informed type's informative messages in this period are effectively truthful, and so correct, reports of the state. This crisis is thus a phenomenon in which a principal dismisses the informed type's correct report and relies only on public information. Remarkably, this crisis is not simply a phenomenon in which the principal substitutes high-quality public information for expertise; rather, high-quality public information sustains this principal's efficient use of the informed type's knowledge.

The informed type's truthfulness arises from her reputation concern. If she were to mix over several informative messages, then these messages must convey the same information and are equivalent reports of the current state. If she were to mix between reporting and misreporting this state, her reputation concern again requires that these reports convey the same information and so, contradictorily, are not informative.

Given the informed type's truthfulness, the crisis occurs because the principal worries that a report of an unlikely state is sent by the uninformed type, who might do so to gamble on the unlikely event of this report turning out to be correct for a high reputation. If public information has sufficiently high quality, then no crisis happens—the principal is quite certain that the uninformed type will not gamble since a report of an unlikely state is too likely to be incorrect and thus to reveal her type. If public information has sufficiently low quality, then no crisis happens either, because the principal considers an expert report as more valuable than public information is in guiding his action. A crisis however occurs if public information has mediocre quality because the principal views the uninformed type as sufficiently likely to gamble and public information as fairly accurate.

1.2 Complementarity

An important insight that emerges from my analysis is a complementarity between the quality of public information and the quality of expert advice: in reputation equilibria satisfying natural conditions, an improvement in public information quality increases not only a principal's utility from dismissing expert advice, but also his utility from following expert advice by mitigating uninformed gambling. Even if better public information could trigger a crisis in view of my main result, it triggers a crisis only because the principal's utility from dismissing expert advice rises faster than his utility from following expert advice does, and also dominates it. Better public information thus unambiguously improves social welfare, defined as the sum of the players' payoffs across the two periods.

This complementarity supports policy advocates concerning the need to equip decision makers with better public information (see, e.g., Howells, 2005), regulations against online misinformation (see, e.g., Funke and Flamini, 2021), and policies such as stress testing to generate public information about financial assets (see, e.g., Hirtle and Lehnert, 2015). While these efforts are typically justified only on the grounds of protecting decision makers who rely on noisy public information, my finding suggests an additional argument of disciplining uninformed gambling; this argument may be particularly relevant in the financial sector where experts often make contrarian calls for career advancement (see, e.g., Zitzewitz, 2001; Bernhardt, Campello, and Kutsoati, 2006; Chen and Jiang, 2006; Bozanic, Chen, and Jung, 2019).

This complementarity also speaks to a recent literature on social media misinformation (e.g., Acemoglu, Ozdaglar, and Siderius, 2022; Mostagir and Siderius, 2022; Chang and Vong, 2022), suggesting a positive externality of more accurate social media contents on the quality of expert advice.

1.3 Related literature

From a modeling standpoint, this paper contributes to the literature on reputational cheap talk (e.g., Ottaviani and Sørensen, 2006a,b). Typical models in this literature do not study decision-making in response to expert advice. Moreover, in these models, an expert speaks to maximize some exogenous payoff function of her reputation. Exogenous reputation payoffs are typically justified as a reduced-form representation of the expert's second-period market wage in a two-period career-concern model such as mine, with a restriction that period-two play is efficient irrespective of past play (see, e.g., Holmström and Costa, 1986; Ottaviani and Sørensen, 2006a,b). My model does not impose this restriction, but rather allows for a systematic analysis of different wages that arise from different equilibrium coordinations; these wages reflect the principals' perceived value of expert advice to their decision-making. In doing so, my analysis characterizes which coordinations are feasible and what their corresponding equilibrium incentives and outcomes are. In particular, my analysis shows that the period-two efficient play restriction could force equilibrium existence to hinge crucially on how off-path beliefs are specified, highlighting the limitations of exogenous reputation payoffs; Section 5 elaborates.[2]

Klein and Mylovanov (2017) also study a model of reputational cheap talk with endogenous market wages rather than with exogenous reputation payoffs. Their analysis has a different objective and features different fundamentals. Their main result is to show that a long horizon can help players achieve the first best outcome in equilibrium in their model but not to characterize different equilibrium outcomes. Their expert does not know her type and does not gamble in equilibrium; the crisis of expertise due to uninformed gambling in my model does not arise in their model.

[2]Exogenous reputation payoffs are also assumed in models other than standard reputational cheap talk models, in part because of their attractive analytical convenience. For example, Prendergast and Stole (1996) study an expert who signals her informativeness via costly investments. Rüdiger and Vigier (2019) study a model in which the public assesses an expert's informativeness based only on endogenous trade outcomes. Smirnov and Starkov (2019) and Shahanaghi (2022) study how experts choose the timing of reports to appear informed when they can report only once and only truthfully.

My analysis complements Morris (2001) and Levy (2004) who propose different reasons for why decision makers dismiss expert advice. Morris (2001) considers a principal who knows that the expert is informed but is unsure if their preferences are aligned. He finds a bad-reputation phenomenon in which an expert whose preference is aligned with that of the principal advises against the principal's interest so as to signal this alignment, making her advice worthless. The crisis in my model is not a bad-reputation phenomenon and, as discussed, is due to uninformed gambling. Levy (2004) studies a model in which a principal, but not the expert, has reputation concerns and could dismiss expert advice to signal her competence.

My model also differs from some reputation models of communication in which all expert type are informed, and an opportunistic type benefits from biasing the principals' actions as well as from pooling with a commitment type who truthfully reports her private information (e.g., Benabou and Laroque, 1992; Mathevet, Pearce, and Stacchetti, 2022). In my model, the expert is possibly uninformed and is impartial about the principals' actions. In addition, there is no commitment type; the informed type's truthfulness is an equilibrium phenomenon. This latter finding might be of independent theoretical interest. A truthful commitment type is often justified as facing rather different payoff consequences than a strategic type does, such as harsh penalties if caught misreporting (see, e.g., Benabou and Laroque, 1992, pp. 926–927). My finding shows how the informed type's reputation concern could lead to her truthfulness when all expert types face identical payoff consequences.

The informed type's equilibrium truthfulness contrasts with a typical view by economists that public information hinders the elicitation of private information, which arises in settings with reputational herding (e.g., Scharfstein and Stein, 1990; Ottaviani and Sørensen, 2001), social learning (e.g., Banerjee, 1992; Bikhchandani, Hirshleifer, and Welch, 1992), bad reputations (e.g., Morris, 2001; Ely and Välimäki, 2003; Maskin and Tirole, 2004; Kartik and Van Weelden, 2019), and strategic complementarity (e.g., Morris and Shin, 2002, 2005; Angeletos and Pavan, 2007).

Finally, my finding that high-quality public information sustains decision makers' efficient use of the informed expert's knowledge speaks to the broad literature on information economics. It complements a key theme in this literature, namely eliciting private information, by highlighting the issue of efficiently utilizing elicited private information.

2 Model

There are two periods, $t = 1, 2$. In each period, an expert (she) sells advice to a new principal (he). This expert has a private type $\theta \in \{I, U\}$: she is an informed type ($\theta = I$) with probability $p \in (0, 1)$ and an uninformed type ($\theta = U$) otherwise.

2.1 Interactions

In each period t, a state $s_t \in S := \{0, 1\}$ is drawn to be 0 with probability $\mu \in [\frac{1}{2}, 1)$ and be 1 otherwise; this draw is independent across time and independent of the type θ. The assumption that $\mu \geq \frac{1}{2}$, i.e., s_t is more likely to be 0, is without loss. The informed type observes the state s_t. The uninformed type and the entering principal do not; thus, they share a prior belief μ that $s_t = 0$. The interpretation is that they form μ based on all relevant public information and μ measures the quality of public information: given a higher μ, the uninformed type and the principal are more convinced that the true state is the one that public information deems most likely, i.e., state 0.

The entering principal hires the expert by paying her a market wage $w_t \in \mathbf{R}_+$ that I specify below in (1). As in the literature on career concerns, he cannot choose to not hire the expert; Appendix I.1 explains that this assumption is innocuous. The expert next sends this principal a message $m_t \in M$, where M is a finite set that is determined in equilibrium. The principal then takes an action $a_t \in S$; this action is hidden from the other players, but my results are unaffected otherwise. Finally, the state s_t is publicly realized and the principal obtains a utility $u(a_t, s_t)$ that is normalized to be one if $a_t = s_t$ and zero otherwise, i.e., $u(a_t, s_t) = \mathbf{1}_{\{a_t = s_t\}}$; symmetry of this utility across states is innocuous. In this period, the expert's payoff is her wage w_t and the principal's payoff is his action utility minus the wage $u(a_t, s_t) - w_t$. For simplicity, there is no discounting; the expert's lifetime payoff is $w_1 + w_2$.

The expert's period-t performance is a pair (m_t, s_t) consisting of her message and the state realization. As in the literature, this performance is publicly observable. In particular, as (1) below makes clear, the expert's period-1 performance affects her period-2 market wage so that this expert faces "career concern" in period 1.

My results extend to richer environments. Appendix I.2 explains that the assumptions of a binary state, an identical and independent state distribution across periods, as well as a common prior state belief μ, merely rule out uninteresting complications. Supplementary Appendix J shows that my results extend if the informed type's state observation is noisy but sufficiently precise, or if the state is sufficiently likely, but not certain, to be realized at the end of each period. Supplementary Appendix L explains that a longer horizon does not affect my insights.

2.2 Histories, beliefs, strategies, and wages

In period 1, the public history is $h_1 := \varnothing$. In period 2, the public history is $h_2 := (m_1, s_1)$, i.e., the expert's period-1 performance. In each period t, the informed type's history $h_t^I := (h_t, s_t)$ consists of the public history and her current state observation; the uninformed type's history h_t^U is plainly the public history h_t. Throughout, I often omit the null history h_1 in the notations.

Given each public history h_t, the entering principal forms a belief p_t, with $p_1 = p$, that the expert is an informed type based on his conjecture of both expert types' strategies; this belief p_t is interpreted as the expert's reputation.

The type-θ expert's strategy is a pair $(\sigma_1^\theta, \sigma_2^\theta)$, where $\sigma_t^\theta(h_t^\theta) \in \Delta(M)$ determines her mixture over period-t messages at history h_t^θ. The period-t principal's strategy is a function $\sigma_t^P(h_t, m_t) \in \Delta(S)$ that determines his mixture over actions given public history h_t and current message m_t. I assume that he takes action 0 if he is indifferent between the two actions; this is without loss as his action is hidden and does not affect future play.[3]

[3]The set of μ given which a principal is indifferent between the two actions has zero measure in my equilibrium analysis. Even if the period-1 principal's action were public, his action would not affect the other players' information about the fundamentals, namely the expert's type and the state, given the expert's period-1 performance. In turn, the tie-breaking assumption would merely rule out the possibility of a measure-zero event in which players coordinate period-2 play based on the period-1 principal's "information-irrelevant" action.

Following the literature on career concerns, market wages are competitive: each principal bids the wage up to a point at which this wage is equal to his perceived marginal benefit from receiving the expert's message. The period-t wage w_t is thus equal to the current principal's expected optimal utility net of his reservation utility. These utilities are computed as follows. Given public history h_t and absent a current message m_t, this principal views action 0 as his optimal action since his state belief (that the current state is 0) is $\mu \geq \frac{1}{2}$. Thus, his reservation utility is μ. Alongside a current message m_t, this principal's optimal utility is $u_t^*(h_t, m_t) := \max_{a \in S} \mathbf{E}[u(a, s_t)|h_t, m_t]$, where this expectation is taken over states s_t given his conjecture of both expert types' strategies. His expected optimal utility at the time of wage payment is thus $\mathbf{E}[u_t^*(h_t, m_t)|h_t]$, where this expectation is taken over messages m_t that he will receive, given his conjecture of both types' strategies. The wage is then

$$w_t \equiv w_t(h_t) := \mathbf{E}[u_t^*(h_t, m_t)|h_t] - \mu. \tag{1}$$

This wage is non-negative because the principal can secure an expected utility of μ by taking action 0. It is zero if the principal anticipates to take action 0 irrespective of the expert's message.

2.3 Equilibrium

The solution concept that I use is weak perfect Bayesian equilibrium, henceforth equilibrium; an equilibrium is weak in the sense that it puts no restriction on off-path beliefs. I focus on equilibria satisfying Property 1 below and refer to these equilibria as *reputation equilibria*.

Property 1. *Given any two period-2 public histories h_2 and $\hat{h}_2$ on path inducing reputations p_2 and $\hat{p}_2$, $w_2(h_2) \geq w_2(\hat{h}_2)$ if and only if $p_2 \geq \hat{p}_2$. In addition, if $p_2 > 0$, then $w_2(h_2) > 0$.*

Note that Property 1 imposes no restriction off path. This property imposes a restriction on the period-2 wage structure on path so that a positive period-2 reputation is valuable to the expert and a higher such reputation is more valuable; various versions of this property appear in the literature, capturing reputation as an asset (e.g., Fudenberg, Levine, and Tirole, 1987; Benabou and Laroque, 1992; Lee and Liu, 2013). This property rules out babbling by the informed type in period 2 on path in which the principal anticipates that the informed type's message conveys none of her private information about the state and so he optimally takes action 0 irrespective of his received message, thereby paying the expert zero wage upfront even if the expert has a positive reputation. Relaxing Property 1 to allow for this latter possibility could be useful in coordinating punishments in richer versions of my model, as I elaborate at the end of Section 6; these punishments are not needed in my main analysis. On the other hand, strengthening Property 1 to additionally require that each expert type's equilibrium lifetime payoff is strictly increasing in her initial reputation p does not affect my results.

3 Main result

In this section, I state my main result. To do so, I adopt two useful definitions from the literature on cheap talk (see, e.g., Sobel, 2013). These definitions apply not only to reputation equilibria, but also more generally to all equilibria. Let $\mathbf{P}$ denote the probability distribution over outcomes in a given equilibrium.[4]

3.1 Informative strategies and non-influential strategies

Definition 1. *In any equilibrium, the informed type's period-t strategy σ_t^I is informative about the state following public history h_t if for some state s_t and some message $m_t \in \operatorname{supp}(\sigma_t^I(h_t, s_t))$ that the informed type sends with positive probability at history $h_t^I = (h_t, s_t)$, the principal's state belief upon receiving m_t differs from his prior state belief, i.e., $\mathbf{P}[s_t = 0 | h_t, m_t] \neq \mu$.*

Lemma 1 below shows that in any reputation equilibrium, if the informed type's period-1 strategy is not informative about the state (following the trivial public history $h_1 = \varnothing$), then this strategy conveys no information about her type either because the uninformed type can and does perfectly pool with her:

[4] An outcome is a tuple $(\theta, (s_1, m_1, a_1), (s_2, m_2, a_2))$, consisting of the type, the states, the messages, and the actions.

Lemma 1. *In any reputation equilibrium, the informed type's period-*1 *strategy* σ_1^I *is not informative about the state if and only if for every state* s_1 *and message* $m_1 \in$ *supp*$(\sigma_1^I(s_1))$ *that the informed type sends with positive probability at history* $h_1^I = s_1$*, the reputation stays put throughout the period:*

$$\mathbf{P}[\theta = I | m_1, s_1] = \mathbf{P}[\theta = I | m_1] = p. \tag{2}$$

In view of Lemma 1, in any reputation equilibrium, I simply say that the informed type's period-1 strategy is informative if it is informative about the state. The proof of Lemma 1 and all other proofs are in the appendices. Appendix A introduces additional notations that are useful in the proofs.

Definition 2. *In any equilibrium, the informed type's period-t strategy* σ_t^I *is non-influential following public history* h_t *if for every state* s_t *and every message* $m_t \in$ supp$(\sigma_t^I(h_t, s_t))$ *that the informed type sends with positive probability at history* $h_t^I =$ (h_t, s_t)*, the current principal's strategy* $\sigma_t^P(h_t, m_t)$ *upon receiving message* m_t *is to take action* 0*. This strategy* σ_t^I *is influential following* h_t *if it is not non-influential.*

Following some public history in equilibrium, if the informed type's strategy is influential so that the current principal takes action 1 upon receiving some informed type's message, then this strategy must be informative. The reason is that the principal optimally takes action 1 only if his belief that the state is 0 falls short of 1/2 and therefore differs from μ. The informed type's strategy could however be informative but non-influential. My main result concerns this latter phenomenon; as motivated at the outset, I interpret this phenomenon as a "crisis of expertise."

3.2 Statement of main result

In any reputation equilibrium, the informed type's period-2 strategy must be influential following every public history h_2 on path. If this is not true at some public history on path, then the informed type has a positive reputation but collects zero wage at this history, violating Property 1. My main result focuses on the informed type's period-1 strategy.

To ease the exposition, in any reputation equilibrium, I say that period 1 is informative if the informed type's period-1 strategy is informative; I also say that play is *efficient* at history h_2 if, following this history, the informed type induces the principal to take an action that matches the state s_2 and the uninformed type induces him to take action 0, i.e., the action that he would have taken if he knew that the expert is uninformed.

My main result is:

Proposition 1. *In each reputation equilibrium with an informative period 1:*

1. *There exists $\bar{\mu} \equiv \bar{\mu}(p) \in (\frac{1}{2}, 1]$ such that for every $\mu \geq \bar{\mu}$, the informed type's (informative) strategy is influential.*
2. *If, in addition, period-2 play is efficient on path, then the informed type's period-1 (informative) strategy is non-influential if and only if $\mu \in C(p)$ for some correspondence $C : (0,1) \rightrightarrows [\frac{1}{2}, 1)$ satisfying:*
   - *(a) If $C(p)$ is nonempty, then $C(p) = [\underline{\mu}, \bar{\mu}]$ for some $\underline{\mu} \equiv \underline{\mu}(p)$ satisfying $\frac{1}{2} < \underline{\mu} \leq \bar{\mu} < 1$, where $\bar{\mu}$ is given in part 1.*
   - *(b) There exists $\bar{p} \in [0,1)$ such that $C(p)$ is nonempty if and only if $p \leq \bar{p}$. For any p, p' satisfying $0 < p < p' \leq \bar{p}$, $C(p') \subsetneq C(p)$.*

A reputation equilibrium with an informative period 1 and efficient period-2 play on path exists.

The basic intuition of Proposition 1 is as described in Section 1.1. Part 1 states that sufficiently high-quality public information sustains the influentiality of the informed type's informative period-1 strategy. In fact, as I shall show, the informed type's message is effectively a truthful report of the state in an informative period 1; part 1 thus shows that high-quality public information sustains the principal's efficient use of the informed type's knowledge.

Part 2 gives a sharp characterization of the period-1 outcome over all public information qualities. Such a characterization requires an assumption on how players coordinate their period-2 play, because this coordination affects the period-2 wage structure and in turn period-1 incentives. Following the tradition of both the literature on reputational cheap talk and the literature on career concerns, part 2 assumes that players coordinate on efficient play at each period-2 history on path.[5] Part 2(a) states that the informed type's period-1 strategy is informative but non-influential if and only if public information quality is mediocre, i.e., neither too close to 1/2 nor to 1. Part 2(b) states that the "crisis" region C of public information qualities over which the informed type's strategy is informative but non-influential is smaller given a higher initial reputation and is empty for sufficiently high reputations.

Indeed, my assumption of efficient period-2 play on path is a weaker version of what is typically assumed in the literature, namely efficient period-2 play irrespective of past play. The standard justification of this latter, stronger assumption is a renegotiation-proof argument that both expert types face no incentive problem in period 2 after collecting the wage. For example, Holmström and Costa (1986, p. 839) writes: "Since the second period is the last in the manager's career, he has no reason not to follow the socially preferred rule, and the firm can trust him to do so." Morris (2001) and Ottaviani and Sørensen (2006a,b) justify this assumption similarly. In Section 5, I show that this stronger assumption is undesirably restrictive in my model. In Section 6, I elaborate on the implications of my assumption of efficient period-2 play on path by systematically exploring all feasible period-2 coordinations from the perspective of maximizing social welfare over the two periods.

The ensuing analysis is structured as follows. In Section 4, I sketch the proof of parts 1 and 2 of Proposition 1; this proof is reported in Appendix C. I do not discuss the proof of existence in the main text; this proof is reported in Appendix D. Then, as mentioned above, I elaborate on the assumption of efficient period-2 play irrespective of past play and my weaker assumption of efficient period-2 play on path in Sections 5 and 6. In Section 7, I report a comparative statics exercise that has direct policy implications, namely the welfare effect of better public information.

[5] My analysis nonetheless also provides a general, implicit characterization of period-1 play over all public information qualities without restricting how players coordinate period-2 play; see Lemma 6.

4 The crisis of expertise

In this section, I sketch the proof of parts 1 and 2 of Proposition 1. In period 1 of any reputation equilibrium, the informed type's strategy is non-influential if and only if the *ex ante* distribution of the principal's actions (which draws action a with probability $\mathbf{P}[a_1 = a]$) draws action 0 with probability one. For this reason, and because both expert types are impartial about the principals' actions, I proceed as follows. Given a reputation equilibrium with an informative period 1, I find a reputation equilibrium with an informative period 1 exhibiting the same period-1 action distribution and a tractable structure of both expert types' strategies; Section 4.1 concerns the informed type and Section 4.2 concerns the uninformed type. Then, in Section 4.3, I derive the period-1 principal's best reply to these strategies, which induces his *ex ante* action distribution.

4.1 The informed type's strategy

I first show that without loss, in an informative period 1 of any reputation equilibrium, messages can be interpreted as state reports and the informed type can be assumed to report the state truthfully.

Lemma 2. *If a reputation equilibrium with an informative period 1 exists, then a reputation equilibrium with an informative period 1 and an identical period-1 action distribution exists in which, in period 1, both expert types' messages are drawn from the set of states S and the informed type reports the true state.*

Take a reputation equilibrium strategy profile, and use it to construct a new strategy profile as follows: relabel all period-1 messages that the informed type sends with positive probability given state 0 as 0, and choose the players' continuation strategies upon message 0 to be identical to those upon one of these relabeled messages. Then, repeat this procedure for the period-1 messages that the informed type sends with positive probability only when the state is 1, but relabel these messages as 1 instead. This new profile constitutes a reputation equilibrium and induces the same period-1 action distribution for the following reason. If the informed type mixes over several messages given some period-1 state, then Property 1 requires that these messages induce the same reputation when the state is publicly realized; these messages thus convey the same information and so induce the same action. The above relabeling of messages and the construction of continuation strategies affect neither the information that these messages convey nor the period-2 wages following public histories that convey this information, and so affects neither the players' incentives nor the period-1 action distribution. In this new equilibrium, period-1 messages are drawn from S because the uninformed type would not send a message that is neither 0 nor 1 to reveal her type.

In the rest of this section, I assume without loss that in reputation equilibria with an informative period 1, period-1 messages are drawn from S and are interpreted as state reports. This finding might be of independent theoretical interest. The assumption that messages are state reports is often imposed as a primitive in reputation models (e.g., Morris, 2001) and is *a priori* restrictive: since the expert cannot commit, the revelation principle does not apply and richer message sets might lead to more equilibrium outcomes. I say that a state report is *correct* if it matches the current state and is *incorrect* otherwise.

The informed type's truthfulness can then be understood as follows. If period 1 is informative and if the informed type were to mix between reporting correctly and incorrectly given some state s_1 in this period, then she must be indifferent between these reports. By Property 1, her period-2 reputations following these reports and the realization of the state s_1 must be equal. These reports must then convey the same, and thus contradictorily no, information about the state s_1. The informed type must then fully reveal the state s_1 in her period-1 report and so, up to relabeling of her reports, she truthfully reports the state s_1. In the rest of this section, I further assume without loss that in reputation equilibria with an informative period 1, the informed type reports the state truthfully, and thus correctly, in period 1.

4.2 The uninformed type's strategy

The structure of the uninformed type's period-1 strategy depends more delicately on the structure of period-2 wages. Section 4.2.1 examines these wages; Section 4.2.2 then turns to the uninformed type's strategy.

4.2.1 Period-2 wages

Lemma 3. *In each reputation equilibrium with an informative period 1, there exists a function* $f : M \times S \to [0,1]$ *such that at each public history* $h_2 = (m_1, s_1)$ *inducing reputation* p_2*, the wage* $w_2(h_2)$ *can be written as* $f(m_1, s_1)\bar{w}(p_2)$*, where*

$$\bar{w}(p_2) := p_2(1-\mu). \tag{3}$$

In any reputation equilibrium, at any history h_2 as stated in the lemma, (3) is an upper bound on the wage $w_2(h_2)$. This bound is attained if the principal's willingness to pay is maximized, namely if period-2 play at history h_2 is efficient. Because efficiency calls for the informed type to induce the principal to take the action that matches the state s_2 and the uninformed type to induce the principal to take action 0, the principal's highest willingness to pay is the expected utility $p_2 + (1-p_2)\mu$ net of his reservation utility μ, i.e., (3). The wage $w_2(h_2)$ is therefore some fraction $f(h_2)$ of (3); this fraction characterizes the value that the principal derives from the expert at history h_2. Efficient play at history h_2 corresponds to $f(h_2) = 1$.

Lemma 4 below examines the structure of this fraction f.

Lemma 4. *In each reputation equilibrium with an informative period 1,* $f(0,0) > 0$*,* $f(1,1) > 0$*, and*

$$f(0,0) > pf(1,1)\frac{1-\mu}{\mu}. \tag{4}$$

If a reputation equilibrium with an informative period 1 exists, then there exists a reputation equilibrium with an informative period 1 and an identical period-1 action distribution in which $f(0,1) = f(1,0) = 0$*.*

In any reputation equilibrium with an informative period 1, any history h_2 featuring a correct period-1 report is on path and induces a positive reputation p_2 because of the informed type's truthfulness in period 1. Property 1 thus ensures that the wage $w_2(h_2) = f(h_2)\bar{w}(p_2)$ is positive and so $f(0,0), f(1,1) > 0$.

To derive (4), the proof of Lemma 4 begins by showing that in an informative period 1, the uninformed type reports 0, but not necessarily 1, with positive probability; the reason is that she considers report 0 as most likely to be correct. The period-2 principal thus expects to see an incorrect report 0, but not necessarily an incorrect report 1, in period 1 on path. An incorrect report 0 thus leads to zero reputation by Bayes' rule and so zero period-2 wage by (1). In period 1 then, because report 0 is correct with probability μ from the uninformed type's perspective, her expected payoff from reporting 0 is strictly smaller than $\mu f(0,0)\bar{w}(1)$; this strictness follows because her reputation on path is strictly smaller than one. Similarly, her expected payoff from reporting 1 in period 1 is at least $(1-\mu)f(1,1)\bar{w}(p)$; her reputation upon a correct report is at least p given the informed type's truthfulness. The uninformed type's incentive constraint to report 0 requires that $\mu f(0,0)\bar{w}(1) > (1-\mu)f(1,1)\bar{w}(p)$; this inequality simplifies to (4). Note that, given the informed type's truthfulness in period 1, efficient period-2 play on path corresponds to $f(0,0) = f(1,1) = 1$—that is, the period-2 principal fully utilizes the expert's value so long as her period-1 report is correct. Note that $f(0,0) = f(1,1) = 1$ satisfies (4).

To facilitate the discussion of the second part of Lemma 4, hereafter I refer to a period-2 play in which both types report the two states equiprobably and the principal (optimally) takes action 0 as a babbling continuation. In this continuation, the wage is zero and no player has a profitable deviation. If the history h_2 features an on-path incorrect period-1 report, then assuming $f(h_2) = 0$ sustained by a babbling continuation does not affect the zero wage at this history and in turn, affects neither the players' incentives nor the action distribution in period 1. If the history h_2 features an off-path incorrect period-1 report instead, then Bayes' rule does not apply. Assuming $f(h_2) = 0$ sustained by a babbling continuation (irrespective of the off-path reputation) nonetheless also affects neither the players' incentives nor the action distribution in period 1: the zero period-2 wage in this continuation strengthens the uninformed type's incentive to not report 1 and the informed type's incentive to report correctly in period 1.

In view of Lemma 4, in the rest of this section, given any reputation equilibrium with an informative period 1 and any public history $h_2 = (m_1, s_1)$ inducing reputation p_2, I write the period-2 wage as $f(m_1, s_1)\bar{w}(p_2)$ and assume without loss that $f(0,1) =$

$f(1,0) = 0$, that $f(0,0), f(1,1) > 0$, and that (4) holds. In general, the function f depends on (p, μ), because the players can condition their play on the exogenous parameters. To emphasize this dependence, I often write $f(h_2)$ as $f(h_2; p, \mu)$.

4.2.2 Uninformed gambling

I now examine the structure of the uninformed type's period-1 strategy.

Lemma 5. *In any reputation equilibrium with an informative period 1 and with $f(0,0;p,\mu) = \gamma_0$ and $f(1,1;p,\mu) = \gamma_1$ for some $\gamma_0, \gamma_1 > 0$, the uninformed type's period-1 strategy is to report 0 with some probability $\alpha^*_{p,\mu}(\gamma_0,\gamma_1)$ that solves*

$$\alpha^*_{p,\mu}(\gamma_0,\gamma_1) \in \arg\max_{\alpha\in[0,1]} \mu\alpha\gamma_0\bar{w}\Bigg(\underbrace{\frac{p}{p+(1-p)\alpha^*_{p,\mu}(\gamma_0,\gamma_1)}}_{\text{term A}}\Bigg) + (1-\mu)(1-\alpha)\gamma_1\bar{w}\Bigg(\underbrace{\frac{p}{p+(1-p)(1-\alpha^*_{p,\mu}(\gamma_0,\gamma_1))}}_{\text{term B}}\Bigg), \tag{5}$$

and to report 1 with complementary probability. This probability $\alpha^*_{p,\mu}$ is:

1. *unique;*
2. *increasing in γ_0 and decreasing in γ_1;*
3. *positive and increasing in μ for fixed (γ_0,γ_1): there exists $\mu^* \equiv \mu^*(p,\gamma_0,\gamma_1) \in (\frac{1}{2},1)$ such that $\alpha^*_{p,\mu}(\gamma_0,\gamma_1)$ is strictly increasing in μ on $[\frac{1}{2},\mu^*)$ and is equal to 1 if $\mu \geq \mu^*$;*
4. *increasing in p for fixed (γ_0,γ_1).*

Figure 1 illustrates $\alpha^*_{p,\mu}(\gamma_0,\gamma_1)$. Condition (5) is a fixed-point characterization. In equilibrium, the uninformed type best replies to the future principal's conjecture of both types' strategies and her best reply matches the conjecture: her reputation upon correctly reporting 0 (resp., 1), given by term A (resp., B) in (5), depends on that principal's conjecture that she reports 0 with probability $\alpha^*_{p,\mu}(\gamma_0,\gamma_1)$ and the informed type reports correctly in period 1. Condition (5) clarifies the uninformed type's desire to improve both the chance of reporting correctly and her reputation upon reporting correctly.

Part 1 reflects the uninformed type's reputation concern. The principals' conjecture of both expert types' strategies must be correct in equilibrium; if the future principal conjectures that this expert reports 0 with a probability that is strictly higher (resp., lower) than $\alpha^*_{p,\mu}(\gamma_0, \gamma_1)$, then her reputation upon correctly reporting 1 (resp., 0), as given by term B (resp., A) in (5), is too high and so she optimally only reports 1 (resp., 0), contrary to the conjecture. In particular, even if the uninformed type views report 0 as more likely to be correct, she might report 1 with positive probability in equilibrium, "gambling" on the unlikely event that this report is correct for a high reputation.

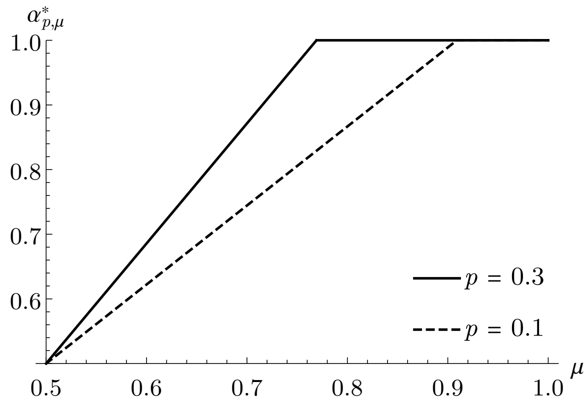
Part 2 reflects the uninformed type's response to future stakes. Given a higher γ_0 (resp., γ_1), the period-2 wage upon a correct report 0 (resp., 1) is higher and so this expert reports 0 with a higher (resp., lower) probability. Part 3 reflects her desire to improve the chance of reporting correctly: $\alpha^*_{p,\mu}(\gamma_0, \gamma_1)$ is positive because, as discussed in Section 4.2.1, the uninformed type views report 0 as most likely to be correct. It is increasing in μ because report 0 is more likely to be correct given a higher μ. This expert does not gamble at all, i.e., $\alpha^*_{p,\mu}(\gamma_0, \gamma_1) = 1$, when μ is sufficiently close to one. Finally, part 4 also reflects the uninformed type's reputation concern: given a higher reputation, she has less gain from gambling and so reports 0 with a higher probability. This finding speaks to, for instance, the empirical evidence in Bozanic et al. (2019) documenting that financial analysts at lower-tier brokerage houses are more likely to make calls contrary to public information for their career advancement.

4.3 The principal's best reply

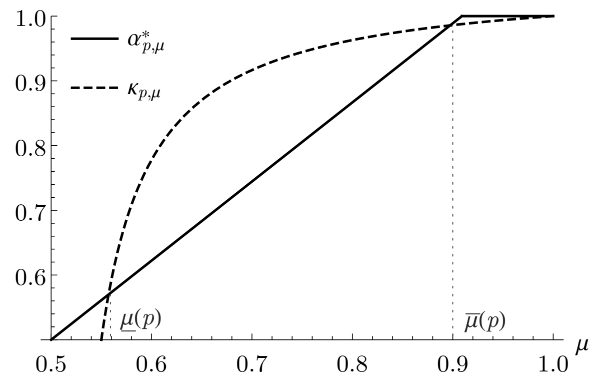
Finally, I derive the period-1 principal's best reply. Define

$$\kappa_{p,\mu} := \frac{\mu(2-p)-1}{(2\mu-1)(1-p)}. \tag{6}$$

Lemma 6. *In any reputation equilibrium with an informative period 1 in which the uninformed type reports* 0 *with probability* $\alpha^*_{p,\mu}(f(0,0;p,\mu), f(1,1;p,\mu))$ *characterized by*



(a) uninformed gambling



(b) principal's best reply ($p = 0.1$)

Figure 1: Period-1 behavior ($\gamma_0 = \gamma_1 = 1$)

(5) *in period 1, the period-1 principal takes action* 0 *irrespective of the expert's report if*

$$\alpha^*_{p,\mu}(f(0,0;p,\mu), f(1,1;p,\mu)) \leq \kappa_{p,\mu} \tag{7}$$

and matches his action with the report otherwise.

The informed type's truthful period-1 strategy could be non-influential because the principal worries that the expert report is sent by the uninformed type—if this principal knew that the expert is informed, he would have matched his action with the expert's report for sure; if he knew that the expert is uninformed, he would have chosen action 0 irrespective of the expert's report. In equilibrium, this principal matches his action with report 0 since this report reinforces the public information that the state is more likely to be 0. This principal however need not match his action with report 1; he takes action 0 despite having received report 1 if the uninformed type is sufficiently likely to gamble, i.e., (7) holds, and matches his action with report 1 otherwise.

Thus, the informed type's period-1 informative strategy is non-influential if and only if (7) holds. Because sufficiently high quality μ of public information disciplines uninformed gambling in view of part 3 of Lemma 5, part 1 of Proposition 1 follows:

Lemma 7. *If $\mu > \bar{\mu}$, where $\bar{\mu}$ is stated in part 1 of Proposition 1, then (7) in Lemma 6 fails so that the informed type's period-1 informative strategy is influential.*

Next, Lemma 8 below imposes the assumption of efficient period-2 play on path to

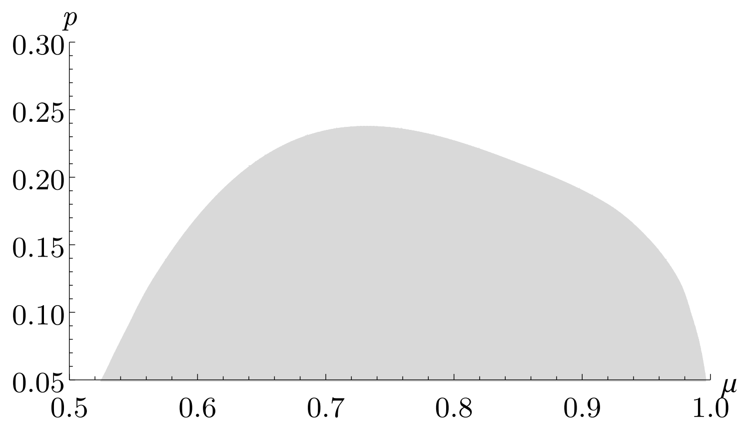


Figure 2: Crisis region $C(p)$

obtain part 2 of Proposition 1:

Lemma 8. *If period-2 play on path is efficient, i.e.,* $f(0,0;p,\mu) = f(1,1;p,\mu) = 1$*, then* (7) *in Lemma 6 simplifies to* $\mu \in C(p)$*, where* C *is stated in Proposition 1 and satisfies part 2 of that proposition, so that the informed type's period-1 informative strategy is non-influential if and only if* $\mu \in C(p)$*.*

Figure 1b illustrates (7) with efficient period-2 play on path and Figure 2 illustrates C. To interpret $C(p)$, suppose that it is nonempty and so $C(p) = [\underline{\mu}, \bar{\mu}]$ as stated in Proposition 1. As discussed, if $\mu > \bar{\mu}$, then the uninformed type is unlikely to gamble and so the principal matches his action with an expert report. If $\mu < \underline{\mu}$ instead, then he also matches his action with an expert report since he considers this report as more valuable than low-quality public information is in guiding his action. In contrast, he takes action 0 irrespective of an expert report if $\mu \in [\underline{\mu}, \bar{\mu}]$, since he views public information as fairly accurate and uninformed gambling as likely to happen. Given a higher reputation p, $C(p)$ is smaller because the principal believes not only that the report is more likely to be sent by the informed type and thus to be correct, but also that, by part 4 of Lemma 5, the uninformed type is less likely to gamble.

5 Market wages and reputation payoffs

My results have been cast in a setting with endogenous market wages but not with an exogenous reputation payoff, although the latter is a common modeling device in the literature on reputational cheap talk. Section 1.3 has mentioned that exogenous reputation payoffs are typically motivated as the expert's period-2 payoff in a two-period career-concern model such as mine, with the restriction of efficient period-2 play irrespective of past play. In each reputation equilibrium, given the function f described in Lemma 3, efficient period-2 play irrespective of past play corresponds to $f(h_2) = 1$ for every history h_2. In turn, the expert's "reputation payoff" is given by the function $\bar{w}(p_2)$ of her period-2 reputation p_2, defined in (3), both on and off path. This contrasts with the weaker assumption of efficient period-2 play on path in Section 4, given which the expert's period-2 payoff is not an exogenous function of her reputation; it is an equilibrium phenomenon and is a function of her period-1 performance—this payoff is zero following an incorrect period-1 report even if this incorrect report induces a positive off-path reputation.

This section highlights some limitations of exogenous reputation payoffs on equilibrium predictions. I show that the assumption of efficient period-2 play irrespective of past play limits the extent to which players can coordinate on punishing incorrect reports and in turn could rule out reputation equilibria with an informative period 1:

Corollary 1. *A reputation equilibrium with an informative period 1 and efficient period-2 play irrespective of past play exists if and only if* $\mu < 1/(1+p)$.

By Lemma 2, if a reputation equilibrium with an informative period 1 exists, then a reputation equilibrium exists in which, in period 1, messages are state reports and the informed type reports the true state. Unlike in Section 4.2 in which each expert type collects a zero period-2 wage upon an incorrect report, efficient period-2 play irrespective of past play here implies that upon an off-path incorrect report, the expert's period-2 wage $\bar{w}(\cdot)$ given in (3) is positive if her (off-path) reputation is positive. If this reputation is too high, then the uninformed type has a profitable deviation to report 1 in an informative period 1; but if the period-2 principal conjectures that this expert reports 1 with positive probability in period 1, the off-path benefit from incorrectly reporting 1 disappears and, provided $\mu \geq 1/(1+p)$ so that report 0 is sufficiently likely to be correct, this expert only reports 0. The uninformed type has no best reply, disrupting the existence of the equilibrium.

If equilibrium refers to perfect Bayesian equilibrium instead of weak perfect Bayesian equilibrium, then even if $\mu \geq 1/(1+p)$, a reputation equilibrium with an informative period 1 exists in which the reputation upon an off-path incorrect report 1 is assumed to be sufficiently small.[6] This latter requirement however conflicts with a typical restriction that this off-path reputation is one in the literature (e.g., Rubinstein, 1985); a typical justification of this restriction, applied to the present context, is as follows. Since the period-2 principal conjectures that the uninformed type only reports 0 in period 1, a report 1 from the expert in this period immediately convinces the principal that the expert is informed before the state is realized.

6 Efficient period-2 play on path

This section assesses my assumption of efficient period-2 play on path in Proposition 1 by exploring all feasible period-2 coordinations in reputation equilibria. Proposition 2 below shows that for a certain range of parameters (p, μ), this assumption constitutes reputation equilibria that maximize social welfare over the two periods and for the other parameters, this assumption reflects the players' failure to coordinate on playing the social optimum. I then interpret Proposition 2 as suggesting that this failure is intuitively appealing, and so the assumption of efficient period-2 play on path is a natural restriction.

I first note that my model allows for a large range of coordination possibilities:

Lemma 9. *Given any function $f : M \times S \to [0,1]$ satisfying $f(1,0) = f(0,1) = 0$, $f(0,0), f(1,1) > 0$, and (4), there exists a reputation equilibrium with an informative period 1 in which messages are drawn from S and the informed type reports the true state, and for each history h_2 inducing reputation p_2, the period-2 wage is $w_2(h_2) = f(h_2)\bar{w}(p_2)$.*

This is a converse of Lemma 3. Intuitively, period 2 is the last period so that given any "target" wage short of $\bar{w}(\cdot)$, a period-2 strategy profile can be constructed to sustain this wage in some reputation equilibrium.

[6] The period-1 strategy profile constructed in Appendix D, coupled with efficient period-2 play irrespective of past play, constitutes a reputation (perfect Bayesian) equilibrium if off-path reputations are assumed to be zero.

Define social welfare in period t as the sum of the players' payoffs in that period, i.e., $w_t + u(a_t, s_t) - w_t$. This welfare simplifies to the principal's action utility $u(a_t, s_t)$.

Definition 3. *A reputation equilibrium is socially optimal if this equilibrium maximizes, among all reputation equilibria, the ex ante sum of social welfare across both periods:*

$$\mathbf{E}\left[u(a_1, s_1) + u(a_2, s_2)\right]. \tag{8}$$

Lemma 10 below is essential.

Lemma 10. *In any socially optimal reputational equilibrium, period 1 is informative. If a reputation equilibrium with an informative period 1 exists, then there exists a reputation equilibrium with identical social welfare* (8) *in which, in period 1, messages are drawn from the set of states S and the informed type truthfully reports the state.*

The social optimum requires that period 1 is informative for the principals to learn about the state and the expert's type. Moreover, the relabeling of messages in this period and the construction of continuation strategies as described in Lemma 2 do not affect (8) and so Lemma 2 applies, giving Lemma 10. In the rest of this section, I assume without loss that in any socially optimal reputation equilibrium, in period 1, the informed type truthfully reports the state and the uninformed type reports state 0 with some probability α. In turn, for the same reason as in Lemma 4, I further assume that $f(1,0) = f(0,1) = 0$.

Proposition 2 characterizes the socially optimal reputation equilibrium outcomes:

Proposition 2. *A socially optimal reputation equilibrium exists. In any socially optimal reputation equilibrium,*

$$(f(0,0), f(1,1), \alpha) := \begin{cases} (1,1,1), & \text{if } \mu \geq \dfrac{1}{1+p}, \\ \left(1, \dfrac{\mu p}{1-\mu}, 1\right), & \text{if } \mu \in \left(\min\left[\dfrac{1+p}{2+p}, \dfrac{1}{1+p}\right], \dfrac{1}{1+p}\right), \\ \left(1, 1, \mu\left(\dfrac{1+p}{1-p}\right) - \dfrac{p}{1-p}\right), & \text{if } \mu \leq \min\left[\dfrac{1+p}{2+p}, \dfrac{1}{1+p}\right]. \end{cases} \tag{9}$$

Proposition 2 shows that the assumption of efficient period-2 play on path constitutes a socially optimal reputation equilibrium if and only if public information quality μ does not take an intermediate value. At the social optimum, $f(0,0) = 1$ since fully utilizing the expert's value upon a correct report 0 improves the period-2 social welfare and also maximizes the reward of a correct report 0, thereby mitigating uninformed gambling and improving the social welfare in period 1.[7] In contrast, $f(1,1)$ need not be one: a higher $f(1,1)$ helps the period-2 principal better utilize the expert's value upon a correct report 1 but increases the reward of a correct report 1, thereby strengthening uninformed gambling in period 1. If public information quality μ is sufficiently high so that no uninformed gambling occurs in period 1 even if $f(1,1) = 1$, the social optimum calls for $f(1,1) = 1$. If μ is sufficiently small, the social optimum also calls for $f(1,1) = 1$ because a correct report 1 is likely to occur. If μ takes an intermediate value instead, the social optimum calls for $f(1,1) < 1$ to preempt uninformed gambling in period 1, i.e., $\alpha = 1$. The set of these intermediate values μ is smaller given a higher reputation p and is empty for high enough reputations, as a correct report is more likely to occur given a higher reputation.

The social optimum (9) that preempts uninformed gambling when μ takes an intermediate value has an implausible feature: if the uninformed type does not gamble in period 1, the informed type would perfectly reveal her type upon a correct report 1 in that period. But then, following this report, the competitive market coordinates to under-utilize this expert's value although she is known to be informed and there is no incentive problem in period 2.[8]

This coordination to preempt gambling also appears at odds with the empirical observations mentioned in Section 1.2. Indeed, this coordination also ensures that the

[7] The principal's utility from matching his action with an expert report is smaller if the uninformed type is more likely to gamble and guide him to take action 1; see (11) below.

[8] This is precisely how Ely and Välimäki (2003) motivate their notion of renegotiation-proofness. Their notion adopted to my setting requires that period-2 play on path is efficient if the expert's reputation is one, which is weaker than requiring efficient period-2 play on path. The reader might wonder how Proposition 2 would change in one were interested in characterizing socially optimal reputation equilibria subject to Ely and Välimäki (2003)'s notion of renegotiation-proofness over the range of intermediate values of μ given which efficient period-2 play on path is not socially optimal. Appendix H shows that their notion precludes the existence of socially optimal reputation equilibria over this range.

informed type's period-1 informative strategy is influential, contradicting the anecdotes that motivate my analysis in the first place:

Corollary 2. *In any socially optimal reputation equilibrium, the informed type's period-1 strategy is influential.*

This corollary shows that non-influentiality of the informed type's informative strategy in period 1 is a socially inefficient phenomenon in reputation equilibria. If the informed type's period-1 strategy is non-influential, then the period-1 social welfare is plainly the principal's reservation utility μ and the expected period-2 social welfare is at most the efficient level $p + (1 - p)\mu$. As Proposition 2 suggests, the players could always coordinate on some $f(1, 1) < 1$ to preempt uninformed gambling in period 1 so that the period-1 social welfare attains the efficient level, as well as coordinate on $f(0, 0), f(1, 1) > 0$ so that the expected period-2 social welfare strictly improves upon the principal's reservation utility.[9]

Supplementary Appendix J.1 studies a noisy perturbation of my model in which the informed type observes a noisy but sufficiently precise signal of the state, but not the state, before sending her message in each period. In that extension, the informed type's truthful report of her signal in period 1 can be incorrect on path; as a result, efficient period-2 play on path limits the extent to which players can coordinate on punishing incorrect reports to sustain the informed type's incentive to truthfully report her signal in period 1. There, to sustain her truthful incentive, the assumption of efficient period-2 play on path and Property 1 are relaxed to allow for an additional coordination possibility: players coordinate on a babbling continuation to punish the expert with zero wage upon an incorrect report, despite the expert having a positive on-path reputation.

7 Better public information and complementarity

This section examines the comparative statics of social welfare (8) with respect to public information quality μ and speaks to the policy discussion in Section 1.2.

[9] Another way to derive this result is to compute directly that α as characterized in (9) is strictly larger than $\kappa_{p,\mu}$ given in (6), and then apply Lemma 6.

The welfare implications of better public information in general depend on how players coordinate their play. In Proposition 3 below, I focus on reputation equilibria satisfying natural conditions concerning the players' coordination and show that in these equilibria, a positive, unambiguous conclusion obtains. In reputation equilibria that satisfy these conditions, Lemma 10 applies and so, as in Section 6, I assume without loss that period-1 messages are state reports, the informed type reports the true state in period 1, and the function f described in Lemma 3 satisfies $f(1,0;p,\mu) = f(0,1;p,\mu) = 0$. Moreover, in period 1, the uninformed type reports 0 with probability $\alpha^*_{p,\mu}(f(0,0;p,\mu), f(1,1;p,\mu))$ that solves (5).

Proposition 3. *In any reputation equilibrium with an informative period 1 in which $f(0,0;p,\mu) = 1$ and $f(1,1;p,\mu)$ is differentiable, satisfying*

$$0 \leq \frac{\partial f(1,1;p,\mu)}{\partial \mu} \leq \frac{f(1,1;p,\mu)}{(1-\mu)\mu}, \tag{10}$$

social welfare (8) is strictly increasing in μ.

Proposition 3 applies to, for example, reputation equilibria with an informative period 1 and efficient period-2 play on path, as well as to socially optimal reputation equilibria. Condition (10) states that in equilibrium, as the quality of public information μ improves, the fraction of the expert's value that the period-2 principal utilizes upon a correct period-1 report 1 increases at a bounded rate. To understand this condition, note that an increase in μ has two opposing effects on the uninformed type's strategy $\alpha^*_{p,\mu}(1, f(1,1;p,\mu))$, as Lemma 5 has highlighted. On the one hand, report 0 is more likely to be correct and so the uninformed type's gambling incentive in period 1 becomes weaker. On the other hand, if the period-2 principal utilizes more value from the expert upon a correct period-1 report, then the period-2 wage upon this report increases and strengthens the uninformed type's gambling incentive in period 1. The bound in (10) ensures that this second effect is dominated by the first, and so better public information unambiguously mitigates uninformed gambling, i.e., $\alpha^*_{p,\mu}(1, f(1,1;p,\mu))$ increases in μ. As is intuitive, given any fixed value $f(1,1;p,\mu)$, the bound on (10) is more constrained if μ is closer to 1/2, in which case public information has lower quality and the period-2 principal values the expert's report more.

In turn, a complementarity between the quality of public information and the quality of expert advice emerges. This latter quality is measured by the principal's utility from matching his action with an expert report, i.e.,

$$u^*_{p,\mu}(1, f(1,1;p,\mu)) := p + (1-p)\Big[\mu\alpha^*_{p,\mu}(1, f(1,1;p,\mu)) \\ + (1-\mu)(1-\alpha^*_{p,\mu}(1, f(1,1;p,\mu)))\Big]. \tag{11}$$

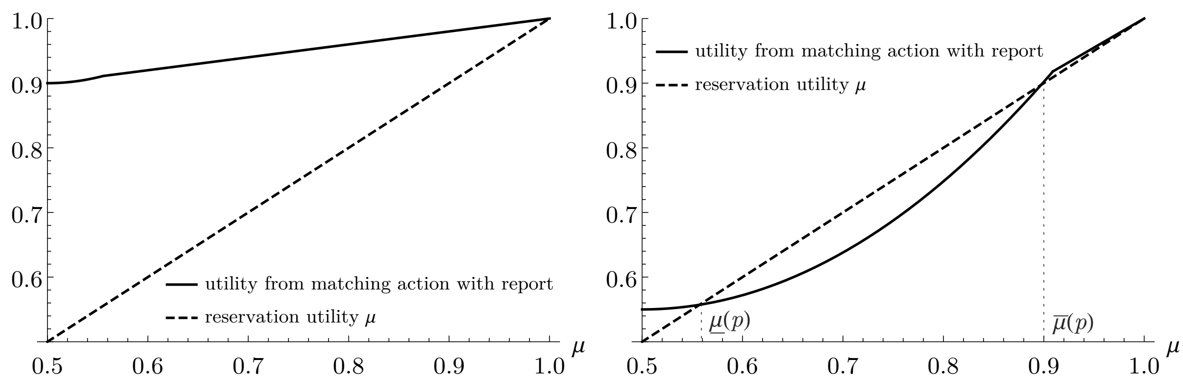
To formally state the complementarity:

Corollary 3. *Suppose that f satisfies the conditions in Proposition 3. Then the utility (11) is strictly increasing in μ.*

Better public information unambiguously mitigates uninformed gambling and so improves (11). In addition, the uninformed type with weaker gambling incentive is more likely to report 0 and better public information ensures that this report is more likely to be correct, further improving (11).

Proposition 3 then follows because better public information unambiguously benefits the principals in both periods. Clearly, it improves their reservation utilities. In addition, in the reputational equilibria that Proposition 3 focuses on, the conditions on f ensure that better public information helps the period-2 principal better utilize the expert's value. Finally, better public information helps the period-1 principal better utilize the expert's value because of the complementarity between the quality of public information and the quality of expert advice.

This complementarity offers an alternative interpretation of the structure of C in Proposition 1. An increase in μ causes the informed type's strategy to be non-influential if the principal's reservation utility μ rises faster than (11) does and also dominates (11). This latter event occurs when both the reputation p and public information quality μ are low, in which case the principal remains quite worried about uninformed gambling despite better public information. Otherwise, this principal views uninformed gambling as negligible, so that (11) dominates his reservation utility. Figure 3 illustrates.



(a) $p = 0.8$ **(b)** $p = 0.1$

Figure 3: Complementarity $(f(0,0;p,\mu) = f(1,1;p,\mu) = 1)$

8 Concluding remarks

I have developed a model of reputational cheap talk to examine "the crisis of expertise." My results identify a non-monotone relationship between the quality of public information and public reliance on expert advice, as well as a novel complementarity between the quality of public information and the quality of expert advice. My model is amenable to sharp comparative statics for policy implications, in particular for assessing the welfare implications of better public information.

On the methodological front, to the best of my knowledge, this paper is the first to systematically analyze market wages and outcomes resulting from different equilibrium coordinations in the literature on reputational cheap talk. Supplementary Appendix J, which examines noisy perturbations of my model, illustrates an additional analytical advantage of working with wages rather than with exogenous reputation payoffs. Noisy environments could make the task of computing beliefs and checking equilibrium incentives daunting, but the flexibility of coordinating wages based on past play lightens this task.

Like standard models in this literature, my analysis addresses applications in which the principals use state realizations to assess the expert's type. It is desirable to extend the present approach to address other applications that feature different information or incentive structures; I leave these issues to future research.

Appendices

A Notations

Throughout, I denote by $\sigma_t^\theta(m|h_t^\theta)$ the probability that type-θ expert sends message m at history h_t^θ given her period-t strategy σ_t^θ. Because h_1 is plainly a null history, I often write $\sigma_1^I(m|h_1, s_1)$ simply as $\sigma_1^I(m|s_1)$ and also write $\sigma_1^U(m|h_1)$ simply as $\sigma_1^U(m)$. In any given equilibrium, I write $\varphi_2(h_2) := \mathbf{P}[\theta = I|h_2]$ as the expert's reputation induced by public history h_2. Given a history h_2 that induces reputation $\varphi_2(h_2) = p_2$, I often write $w_2(p_2; h_2)$ instead of $w_2(h_2)$ to stress the dependence of the period-2 wage on p_2. Finally, given a state $s \in \{0, 1\}$, let $\neg s$ denote the other state.

B Proof of Lemma 1

Fix a reputation equilibrium, with strategy profile $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$. The following claim is useful.

Claim 1. *It holds that*

$$\mathrm{supp}(\sigma_1^U) \subseteq \bigcup_{s_1=0,1} \mathrm{supp}(\sigma_1^I(s_1)). \tag{12}$$

Proof of Claim 1. Suppose towards a contradiction that the uninformed type's strategy σ_1^U draws a message $m_1' \notin \bigcup_{s_1=0,1} \mathrm{supp}(\sigma_1^I(s_1))$ with positive probability. Her payoff is

$$\begin{aligned} & w_1 + \sum_{m_1 \in \mathrm{supp}(\sigma_1^U)} [\mu w_2(m_1, 0) + (1-\mu) w_2(m_1, 1)] \\ = {} & w_1 + \sum_{m_1 \in \mathrm{supp}(\sigma_1^U) \setminus \{m_1'\}} [\mu w_2(m_1, 0) + (1-\mu) w_2(m_1, 1)], \end{aligned}$$

where the second line follows because history $h_2 = (m_1', 0)$ induces $p_2 = 0$ and so, by (1), $w_2(m_1', 0) = 0$. Consider a deviation by the uninformed type to some strategy $(\tilde{\sigma}_1^U, \sigma_2^U)$ in which $\tilde{\sigma}_1^U$ is identical to σ_1^U except that m_1' is relabeled as some $\tilde{m}_1 \in \mathrm{supp}(\sigma_1^I(0))$.

The uninformed type's payoff from this deviation is

$$\begin{aligned}&w_1 + \sum_{m_1 \in \text{supp}(\tilde{\sigma}_1^U)} [\mu w_2(m_1, 0) + (1-\mu) w_2(m_1, 1)] \\ &= w_1 + \sum_{m_1 \in \text{supp}(\sigma_1^U) \setminus \{m_1'\}} [\mu w_2(m_1, 0) + (1-\mu) w_2(m_1, 1)] + [\mu w_2(\tilde{m}_1, 0) + (1-\mu) w_2(\tilde{m}_1, 1)] .\end{aligned}$$

Since $\tilde{m}_1 \in \text{supp}(\sigma_1^I(0))$, history $(\tilde{m}_1, 0)$ induces $p_2 > 0$. By Property 1, $w_2(\tilde{m}_1, 0) > 0$. Thus, this deviation is profitable. Contradiction. ■

I prove the two directions of the lemma in order. Suppose first that period 1 is not informative. Fix $m \in \bigcup_{s_1=0,1} \text{supp}(\sigma_1^I(s_1))$. By Definition 1 and by Bayes' rule,

$$\mu = \mathbf{P}[s_1 = 0 | m_1 = m] = \frac{\mu(p\sigma_1^I(m|0) + (1-p)\sigma_1^U(m))}{\mu(p\sigma_1^I(m|0) + (1-p)\sigma_1^U(m)) + (1-\mu)(p\sigma_1^I(m|1) + (1-p)\sigma_1^U(m))}. \tag{13}$$

This condition requires that

$$\sigma_1^I(m|0) = \sigma_1^I(m|1). \tag{14}$$

Since $m \in \bigcup_{s_1=0,1} \text{supp}(\sigma_1^I(s_1))$, either $\sigma_1^I(m|0) > 0$ or $\sigma_1^I(m|1) > 0$. (14) then implies that $\sigma_1^I(m|0) > 0$ and $\sigma_1^I(m|1) > 0$. Consider two cases.

1. Suppose that $\sigma_1^I(m|0) = 1$. By (14), $\sigma_1^I(m|1) = 1$ and so, by (12), $\text{supp}(\sigma_1^U) = \{m\}$. As a result, (2) follows:

$$\mathbf{P}[\theta = I | m_1 = m, s_1 = 0] = \frac{p\sigma_1^I(m|0)}{p\sigma_1^I(m|0) + (1-p)\sigma_1^U(m)} = p, \tag{15}$$

$$\mathbf{P}[\theta = I | m_1 = m, s_1 = 1] = \frac{p\sigma_1^I(m|1)}{p\sigma_1^I(m|1) + (1-p)\sigma_1^U(m)} = p, \tag{16}$$

$$\mathbf{P}[\theta = I | m_1 = m] = \frac{p(\mu\sigma_1^I(m|0) + (1-\mu)\sigma_1^I(m|1))}{p(\mu\sigma_1^I(m|0) + (1-\mu)\sigma_1^I(m|1)) + (1-p)\sigma_1^U(m)} = p. \tag{17}$$

2. Suppose that $\sigma_1^I(m|0) \in (0, 1)$. Let $\text{supp}(\sigma_1^I(0)) = \{m^1, \ldots, m^n\}$ in which $n > 1$ and, without loss, $m^1 = m$. The informed type must be indifferent among all messages in $\text{supp}(\sigma_1^I(0))$ at history $h_1^I = s_1 = 0$. By Property 1, this indifference

means that there is some $p^\dagger \in [0,1]$ such that

$$\varphi_2(m^k, 0) = \frac{p\sigma_1^I(m^k|0)}{p\sigma_1^I(m^k|0) + (1-p)\sigma_1^U(m^k)} = p^\dagger, \quad \forall k = 1, \dots, n. \tag{18}$$

Observe that $p = p^\dagger$. This is because (18) implies that

$$p^\dagger = \frac{p\sum_{k=1}^n \sigma_1^I(m^k|0)}{p\sum_{k=1}^n \sigma_1^I(m^k|0) + (1-p)\sum_{k=1}^n \sigma_1^U(m^k)} = p.$$

In turn, (18) yields $\sigma_1^I(m^k|0) = \sigma_1^U(m^k)$ for each $k = 1, \dots, n$. By (14), $\sigma_1^I(m^k|1) = \sigma^U(m^k)$ and so, (15)—(17) hold with $m_1 = m^k$ for each $k = 1, \dots, n$.

This proves one direction of the lemma. To prove the converse, fix s_1 and suppose that (2) holds for each $m \in \text{supp}(\sigma_1^I(s_1))$. Take one such m. If $s_1 = 0$, then (15) implies that $\sigma_1^I(m|0) = \sigma_1^U(m)$ and in turn, by (17), $\sigma_1^I(m|1) = \sigma_1^U(m) = \sigma_1^I(m|0)$ and so (13) holds. If $s_1 = 1$ instead, an identical argument applies.

C Proof of parts 1 and 2 of Proposition 1

C.1 Proof of Lemma 2

Fix a reputation equilibrium with an informative period 1. Let $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$ denote the profile of strategies in this equilibrium, and let M denote the set of equilibrium messages. Relabeling the messages if necessary, assume without loss that $S \cap M = \varnothing$. Fix $s_1 = s$. Suppose that there are two messages $m, m' \in M$ given which $\sigma_1^I(m|s) > 0$ and $\sigma_1^I(m'|s) > 0$. The informed type must be indifferent between sending these two messages at history $h_1^I = s$. Property 1 then implies that $\varphi_2(m, s) = \varphi_2(m', s)$, i.e.,

$$\frac{p\sigma_1^I(m|s)}{p\sigma_1^I(m|s) + (1-p)\sigma_1^U(m)} = \frac{p\sigma_1^I(m'|s)}{p\sigma_1^I(m'|s) + (1-p)\sigma_1^U(m')}. \tag{19}$$

I begin by showing a preliminary result that the following new strategy profile $\tilde{\sigma} := (\tilde{\sigma}_t^I, \tilde{\sigma}_t^U, \tilde{\sigma}_t^P)_{t=1,2}$ constitutes a reputation equilibrium that induces the same period-1 action distribution as the present equilibrium does. This profile $\tilde{\sigma}$ is identical to σ

except that $m, m' \in \bigcup_\theta \bigcup_{h_1^\theta} \text{supp}\sigma_1^\theta(h_1^\theta)$ are relabeled as s in $\bigcup_\theta \bigcup_{h_1^\theta} \text{supp}\tilde{\sigma}_1^\theta(h_1^\theta)$ and

$$(\tilde{\sigma}_2^I(s,s,\cdot), \tilde{\sigma}_2^U(s,s), \tilde{\sigma}_2^P(s,s)) := (\sigma_2^I(m,s,\cdot), \sigma_2^U(m,s), \sigma_2^P(m,s)),$$
$$(\tilde{\sigma}_2^I(s,\neg s,\cdot), \tilde{\sigma}_2^U(s,\neg s), \tilde{\sigma}_2^P(s,\neg s)) := (\sigma_2^I(m,\neg s,\cdot), \sigma_2^U(m,\neg s), \sigma_2^P(m,\neg s)).$$

Let $\tilde{\mathbf{P}}$ and $\tilde{\varphi}_2(\cdot)$ denote the counterparts of $\mathbf{P}$ and $\varphi_2(\cdot)$ induced by the new strategy profile. Consider two cases.

1. Suppose that $\sigma_1^U(m) > 0$ or $\sigma_1^U(m') > 0$. Since m and m' are arbitrarily picked, assume without loss that $\sigma_1^U(m) > 0$. I proceed via three claims.

   Claim 2. $\tilde{\varphi}_2(s,s) = \varphi_2(m,s) = \varphi_2(m',s)$ *and* $\tilde{\varphi}_2(s,\neg s) = \varphi_2(m,\neg s) = \varphi_2(m',\neg s)$.

   Proof of Claim 2. By (19), $\sigma_1^U(m) > 0$ implies $\sigma_1^U(m') > 0$. The uninformed type must then be indifferent between sending m and m' in period 1:

$$\begin{aligned} &\mathbf{P}[s_1 = s] \cdot w_2\left(\varphi(m,s); m, s\right) + \mathbf{P}[s_1 \neq s] \cdot w_2\left(\varphi(m,\neg s); m, \neg s\right) \\ &\quad = \mathbf{P}[s_1 = s] \cdot w_2\left(\varphi(m',s); m', s\right) + \mathbf{P}[s_1 \neq s] \cdot w_2\left(\varphi(m',\neg s); m', \neg s\right). \end{aligned} \tag{20}$$

   By (19) and by Property 1, (20) simplifies to $\varphi(m,\neg s) = \varphi(m',\neg s)$, i.e.,

$$\frac{p\sigma_1^I(m|\neg s)}{p\sigma_1^I(m|\neg s) + (1-p)\sigma_1^U(m)} = \frac{p\sigma_1^I(m'|\neg s)}{p\sigma_1^I(m'|\neg s) + (1-p)\sigma_1^U(m')}. \tag{21}$$

   The claim then follows by observing that

$$\begin{aligned} \tilde{\varphi}_2(s,s) &= \frac{p\tilde{\sigma}_1^I(s|s)}{p\tilde{\sigma}_1^I(s|s) + (1-p)\tilde{\sigma}_1^U(s)} \\ &= \frac{p[\sigma_1^I(m|s) + \sigma_1^I(m'|s)]}{p[\sigma_1^I(m|s) + \sigma_1^I(m'|s)] + (1-p)[\sigma_1^U(m) + \sigma_1^U(m')]} \\ &= \frac{p\sigma_1^I(m|s)}{p\sigma_1^I(m|s) + (1-p)\sigma_1^U(m)} = \frac{p\sigma_1^I(m'|s)}{p\sigma_1^I(m'|s) + (1-p)\sigma_1^U(m')} \\ &= \varphi_2(m,s) = \varphi_2(m',s), \end{aligned} \tag{22}$$

where the second last line follows from (19) and similarly, by (21),

$$
\begin{aligned}
\tilde{\varphi}_2(s,\neg s) &= \frac{p\tilde{\sigma}_1^I(s|\neg s)}{p\tilde{\sigma}_1^I(s|\neg s)+(1-p)\tilde{\sigma}_1^U(s)} \\
&= \frac{p\sigma_1^I(m|\neg s)}{p\sigma_1^I(m|\neg s)+(1-p)\sigma_1^U(m)} = \frac{p\sigma_1^I(m'|\neg s)}{p\sigma_1^I(m'|\neg s)+(1-p)\sigma_1^U(m')} \\
&= \varphi_2(m,\neg s) = \varphi_2(m',\neg s).
\end{aligned} \tag{23}
$$

■

Claim 3. *$\tilde{\sigma}$ constitutes a reputation equilibrium.*

Proof of Claim 3. Let $\tilde{w}_2(h_2)$ denote the counterpart of $w_2(h_2)$ given $\tilde{\sigma}$. By construction of $\tilde{\sigma}$ and by Property 1, $\tilde{w}_2(s,s) = w_2(m,s) = w_2(m',s)$ and $\tilde{w}_2(s,\neg s) = w_2(m,\neg s) = w_2(m',\neg s)$, and for every history $h_2 \notin \{(s,s),(s,\neg s)\}$, $\tilde{w}_2(h_2) = w_2(h_2)$. Because σ constitutes an equilibrium and so no expert type has a profitable deviation from σ in period 1, neither type has a profitable deviation from $\tilde{\sigma}$ in period 1. The period-1 principal also has no profitable deviation from $\tilde{\sigma}$. This is because this principal has no profitable deviation from σ_1^P, $\tilde{\sigma}_1^P = \sigma_1^P$ by construction, and his state belief upon receiving message s given $\tilde{\sigma}$ is equal to that upon receiving message m or m' given σ:

$$
\begin{aligned}
&\tilde{\mathbf{P}}[s_1 = s|m_1 = s] \\
&= \frac{(p\tilde{\sigma}_1^I(s|s)+(1-p)\tilde{\sigma}_1^U(s))\tilde{\mathbf{P}}[s_1=s]}{(p\tilde{\sigma}_1^I(s|s)+(1-p)\tilde{\sigma}_1^U(s))\tilde{\mathbf{P}}[s_1=s]+(p\tilde{\sigma}_1^I(s|\neg s)+(1-p)\tilde{\sigma}_1^U(s))\tilde{\mathbf{P}}[s_1=\neg s]}
\end{aligned} \tag{24}
$$

$$
\begin{aligned}
&= \frac{\mathbf{P}[s_1=s]}{\mathbf{P}[s_1=s]+\dfrac{p\sigma_1^I(m|\neg s)+(1-p)\sigma_1^U(m)}{p\sigma_1^I(m|s)+(1-p)\sigma_1^U(m)}\mathbf{P}[s_1=\neg s]} \\
&= \mathbf{P}[s_1=s|m_1=m] = \mathbf{P}[s_1=s|m_1=m'],
\end{aligned} \tag{25}
$$

where the last two lines follow from (22) and (23), and $\tilde{\mathbf{P}}[s_1 = s] = \mathbf{P}[s_1 = s]$ and $\tilde{\mathbf{P}}[s_1 = \neg s] = \mathbf{P}[s_1 = \neg s]$. Because period 2 is the last period, neither expert type has a profitable deviation in period 2. Finally, because σ constitutes a reputation equilibrium, by construction of $\tilde{\sigma}$, the period-2 principal has no profitable deviation from $\tilde{\sigma}$ and so the claim follows. ■

Claim 4. *$\tilde{\sigma}$ induces the same period-1 action distribution as σ does.*

Proof of Claim 4. Let $\tilde{M}$ denote the set of messages in the equilibrium given $\tilde{\sigma}$. For each action a,

$$
\begin{aligned}
\tilde{\mathbf{P}}[a_1 = a] &= \sum_{\tilde{m}\in\tilde{M}} \tilde{\mathbf{P}}[m_1 = \tilde{m}]\tilde{\mathbf{P}}[a_1 = a|m_1 = \tilde{m}] \\
&= \sum_{\tilde{m}\in\tilde{M}\setminus\{s\}} \tilde{\mathbf{P}}[m_1 = \tilde{m}]\tilde{\mathbf{P}}[a_1 = a|m_1 = \tilde{m}] + \tilde{\mathbf{P}}[m_1 = s]\tilde{\mathbf{P}}[a_1 = a|m_1 = s] \\
&= \sum_{\tilde{m}\in\tilde{M}\setminus\{s\}} \Big(\tilde{\mathbf{P}}[m_1 = \tilde{m}]\tilde{\mathbf{P}}[a_1 = a|m_1 = \tilde{m}] \\
&\qquad + (\mathbf{P}[m_1 = m] + \mathbf{P}[m_1 = m'])\mathbf{P}[a_1 = a|m_1 = m]\Big) \\
&= \sum_{\tilde{m}\in M\setminus\{m,m'\}} \Big(\mathbf{P}[m_1 = \tilde{m}]\mathbf{P}[a_1 = a|m_1 = \tilde{m}] \\
&\qquad + \mathbf{P}[m_1 = m]\mathbf{P}[a_1 = a|m_1 = m] + \mathbf{P}[m_1 = m']\mathbf{P}[a_1 = a|m_1 = m]\Big) \\
&= \sum_{\tilde{m}\in M\setminus\{m,m'\}} \Big(\mathbf{P}[m_1 = \tilde{m}]\mathbf{P}[a_1 = a|m_1 = \tilde{m}] \\
&\qquad + \mathbf{P}[m_1 = m]\mathbf{P}[a_1 = a|m_1 = m] + \mathbf{P}[m_1 = m']\mathbf{P}[a_1 = a|m_1 = m']\Big) \\
&= \sum_{\tilde{m}\in M} \mathbf{P}[m_1 = \tilde{m}]\mathbf{P}[a_1 = a|m_1 = \tilde{m}] = \mathbf{P}[a_1 = a].
\end{aligned}
$$

The third line holds by construction of $\tilde{\sigma}$ and the fifth holds by (25). ■

2. Suppose that $\sigma_1^U(m) = \sigma_1^U(m') = 0$. I proceed via two claims.

Claim 5. $\sigma_1^I(m|\neg s) = \sigma_1^I(m'|\neg s) = 0$.

Proof of Claim 5. Suppose, towards a contradiction, that $\sigma_1^I(m|\neg s) > 0$. Then, $\varphi(m, s) = \varphi(m, \neg s) = 1$. Because $\sigma_1^I(m|s) > 0$ and $\sigma_1^U(m) > 0$ by assumption, the uninformed type can secure an expected period-2 wage $\mathbf{P}[s_1 = s]w_2(1; m, s) + \mathbf{P}[s_1 = \neg s]w_2(1; m, \neg s)$ by deviating to send m in period 1. This wage is her highest possible period-2 payoff by Property 1. Thus, in period 1, the uninformed type must send some other message $\hat{m}$ on path that attains this highest period-2 payoff, but achieving reputation one in this payoff requires $\sigma_1^U(\hat{m}) = 0$ by Bayes' rule. Contradiction. Thus, $\sigma^I(m'|s) = 0$. The proof of $\sigma^I(m'|\neg s) = 0$ is identical. ■

Claim 6. *$\tilde{\sigma}$ constitutes a reputation equilibrium and induces the same period-1 action distribution as σ does.*

Proof of Claim 6. By Claim 5, histories $(m, \neg s)$ and $(m', \neg s)$ are off path given σ. By construction of $\tilde{\sigma}$, history $(s, \neg s)$ is off path given $\tilde{\sigma}$. Moreover, because $\sigma_1^U(m) = \sigma_1^U(m') = 0$ implies that $\tilde{\sigma}^U(s) = 0$ by construction of $\tilde{\sigma}$, $\tilde{\varphi}(s, s) = \varphi(m, s) = \varphi(m', s) = 1$. The remainder of this proof is identical to the proofs of Claims 3 and 4. ■

Now, construct a new strategy profile $\hat{\sigma} := (\hat{\sigma}_t^I, \hat{\sigma}_t^U, \hat{\sigma}_t^P)_{t=1,2}$ from σ as follows, which constitutes a reputation equilibrium with an identical period-1 action distribution in which both types' messages are drawn from S. Relabel all messages $m \in \text{supp}(\sigma_1^I(0))$ as 0 so that

$$\hat{\sigma}_1^I(0|0) = \sum_{m \in \text{supp}(\sigma_1^I(0))} \sigma_1^I(m|0), \quad \text{and} \quad \hat{\sigma}_1^U(0) = \sum_{m \in \text{supp}(\sigma_1^I(0)) \cap \text{supp}(\sigma_1^U)} \sigma_1^U(m).$$

Define, for some $m' \in \text{supp}(\sigma_1^I(0))$ and for each $s \in S$,

$$(\hat{\sigma}_2^I(0, s, \cdot), \hat{\sigma}_2^U(0, s), \hat{\sigma}_2^P(0, s)) := (\sigma_2^I(m', s, \cdot), \sigma_2^U(m', s), \sigma_2^P(m', s)).$$

Then, relabel all messages $m \in \text{supp}(\sigma_1^I(1)) \setminus \text{supp}(\sigma_1^I(0))$ as 1, so that

$$\hat{\sigma}_1^I(1|1) = \sum_{m \in \text{supp}(\sigma_1^I(1)) \setminus \text{supp}(\sigma_1^I(0))} \sigma_1^I(m|1),$$
$$\hat{\sigma}_1^U(1) = \sum_{m \in (\text{supp}(\sigma_1^I(1)) \setminus \text{supp}(\sigma_1^I(0))) \cap \text{supp}(\sigma_1^U)} \sigma_1^U(m).$$

Define, for some $m' \in \text{supp}(\sigma_1^I(1)) \setminus \text{supp}(\sigma_1^I(0))$ and for each $s \in S$,

$$(\hat{\sigma}_2^I(1, s, \cdot), \hat{\sigma}_2^U(1, s), \hat{\sigma}_2^P(1, s)) := (\sigma_2^I(m', s, \cdot), \sigma_2^U(m', s), \sigma_2^P(m', s)).$$

By construction and by Claim 1, $\bigcup_{s=0,1} \text{supp}(\hat{\sigma}_1^I(s)) \cup \text{supp}(\hat{\sigma}_1^U) = S$. An iterative application of Claims 2—6 above yields that this new strategy profile $\hat{\sigma}$ constitutes a reputation equilibrium that induces the same period-1 action distribution as σ does.

Next, I turn to the informed type's truthfulness in period 1. Fix a reputation equilibrium with an informative period 1 and with strategy profile $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$, and assume that both expert types draw their messages from S in period 1. Define $\tau_s := \sigma_1^I(s|s)$ and $\alpha := \sigma_1^U(0)$.

Claim 7. *It holds that*

$$\tau_0 \neq 1 - \tau_1, \tag{26}$$

$$\textit{and} \qquad \tau_0 \neq \alpha \quad \textit{or} \quad \tau_1 \neq 1 - \alpha. \tag{27}$$

Proof of Claim 7. By Definition 1,

$$\mu \neq \mathbf{P}[s_1 = 0 | m_1 = 0] = \frac{\mu(p\tau_0 + (1-p)\alpha)}{\mu(p\tau_0 + (1-p)\alpha) + (1-\mu)(p(1-\tau_1) + (1-p)\alpha)},$$

or

$$\mu \neq \mathbf{P}[s_1 = 0 | m_1 = 1] = \frac{\mu(p(1-\tau_0) + (1-p)(1-\alpha))}{\mu(p(1-\tau_0) + (1-p)(1-\alpha)) + (1-\mu)(p\tau_1 + (1-p)(1-\alpha))}.$$

Either expression requires (26) and (27). ■

Claim 8. *Either* $\tau_0 = \tau_1 = 1$ *or* $\tau_0 = \tau_1 = 0$*.*

Proof of Claim 8. I first show that $\tau_0, \tau_1 \in \{0, 1\}$. Suppose, towards a contradiction, that $\tau_0 \in (0, 1)$, then the informed type must be indifferent between reporting 0 or 1 given state 0. This and Property 1 imply:

$$\varphi(0,0) = \varphi(1,0) \implies \frac{p\tau_0}{p\tau_0 + (1-p)\alpha} = \frac{p(1-\tau_0)}{p(1-\tau_0) + (1-p)(1-\alpha)}. \tag{28}$$

This equation simplifies to $\tau_0 = \alpha$, and so $\varphi(0,0) = \varphi(1,0)$. (27) then implies that $\tau_1 \neq 1 - \alpha$. Because $\tau_0 \in (0,1)$ and $\tau_0 = \alpha$, $\alpha \in (0,1)$ and so the uninformed type must be indifferent between reporting the two states:

$$\mu w_2\left(\varphi(0,0); 0, 0\right) + (1-\mu) w_2\left(\varphi(0,1); 0, 1\right) = \mu w_2\left(\varphi(1,0); 1, 0\right) + (1-\mu) w_2\left(\varphi(1,1); 1, 1\right).$$

Because $\varphi(0,0) = \varphi(1,0)$, Property 1 implies that this equation simplifies to $w_2\left(\varphi(0,1); 0, 1\right) = w_2\left(\varphi(1,1); 1, 1\right)$ which, by Property 1, further simplifies to $\varphi(0,1) = \varphi(1,1)$ and so $\tau_1 = 1 - \alpha$. Contradiction. Thus, $\tau_0 \in \{0, 1\}$ and analogously, $\tau_1 \in \{0, 1\}$. Finally, by (26), Claim 8 follows. ■

If $\tau_0 = \tau_1 = 0$, then relabel report 0 as report 1 and relabel report 1 as report 0. Since this relabeling does not affect incentives, it produces a reputation equilibrium in which $\tau_0 = \tau_1 = 1$ and the period-1 action distribution is identical.

C.2 Proof of Lemma 3

Fix a reputation equilibrium and a period-2 public history h_2 inducing reputation p_2. Let $\lambda^I(a|h_2, s_2)$ denote the probability that the principal takes action a conditional on an informed type and history $h_2^I = (h_2, s_2)$; let $\lambda^U(h_2)$ denote the probability that the principal takes action 0 conditional on an uninformed type and history h_2. At history h_2, the period-2 principal's expected utility is

$$\begin{aligned}
&p_2(\mu\lambda^I(0|h_2,0) + (1-\mu)\lambda^I(1|h_2,1)) + (1-p)(\mu\lambda^U(h_2) + (1-\mu)(1-\lambda^U(h_2))) \\
&\leq p_2(\mu + 1 - \mu) + (1-p_2)\mu = p_2 + (1-p_2)\mu.
\end{aligned}$$

An upper bound on the expert's period-2 wage is thus $p_2 + (1-p_2)\mu - \mu = \bar{w}(p_2)$.

C.3 Proof of Lemma 4

Fix a reputation equilibrium with an informative period 1 and with strategy profile $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$. Define $\alpha := \sigma_1^U(0)$.

Claim 9. $\alpha > 0$.

Proof of Claim 9. Suppose, towards a contradiction, that $\alpha = 0$. The uninformed type's payoff from reporting 1 must exceed that from reporting 0:

$$(1-\mu)w_2(p; 1, 1) \geq (1-\mu)w_2(\hat{p}; 0, 1) + \mu w_2(1; 0, 0), \tag{29}$$

where $\hat{p}$ denotes the off-path reputation upon an incorrect report 0 given the market's conjecture that $\alpha = 0$. But (29) violates Property 1. Contradiction. ■

By Claim 9, the arguments for why $f(0,0), f(1,1) > 0$ and why (4) holds are clear from the main text. Next, by Claim 9 and by Bayes' rule, $\varphi(1,0) = 0$ and by (1), $w_2(0; 1, 0) = 0$. The uninformed type's incentive constraint in period 1 is therefore:

$$\alpha = 1 \implies \mu w_2(p; 0, 0) \geq \mu w_2(\hat{p}'; 1, 0) + (1-\mu)w_2(1; 1, 1), \tag{30}$$

$$\alpha \in (0,1) \implies \mu w_2\left(\frac{p}{p + (1-p)\alpha}; 0, 0\right) = (1-\mu)w_2\left(\frac{p}{p + (1-p)(1-\alpha)}; 1, 1\right). \tag{31}$$

where $\hat{p}'$ in (30) denotes the off-path reputation upon an incorrect report 1 when the market conjectures that $\alpha = 1$. If this constraint holds, then it continues to hold with $w_2(\hat{p}'; 1, 0) = 0$ in (30) and (trivially) in (31).

The informed type's incentive constraint is

$$s_1 = 0 \quad \Longrightarrow \quad w_2\left(\frac{p}{p+(1-p)\alpha}; 0, 0\right) \geq w_2(\hat{p}'; 1, 0),$$
$$s_1 = 1 \quad \Longrightarrow \quad w_2\left(\frac{p}{p+(1-p)(1-\alpha)}; 1, 1\right) \geq 0.$$

If this constraint holds, then it continues to hold with $w_2(\hat{p}'; 1, 0) = 0$.

Since the principals play their myopic best replies to both types' strategies, the above arguments ensure that there exists an reputation equilibrium with an identical period-1 action distribution and with $w_2(\cdot; 0, 1) = w_2(\cdot; 1, 0) = 0$, in which both these wages are sustained by babbling. Thus, it is without loss to set $f(0, 1) = f(1, 0) = 0$.

C.4 Proof of Lemma 5

The unique solution to (5) is

$$\alpha^*_{p,\mu}(\gamma_0, \gamma_1) := \begin{cases} \dfrac{(\gamma_0 + \gamma_1 p)\mu - \gamma_1 p}{(1-p)(\gamma_1 + \mu(\gamma_0 - \gamma_1))}, & \text{if } \mu \leq \dfrac{\gamma_1}{\gamma_0 p + \gamma_1}, \\ 1, & \text{otherwise.} \end{cases} \tag{32}$$

This shows part 1. Direct calculations verify that this solution satisfies parts 2—4.

C.5 Proof of Lemma 6

The principal optimally matches his action with report 0 if and only if his payoff from doing so exceeds that from mismatching the report:

$$\begin{aligned} \frac{p\mu}{p\mu + (1-p)\alpha^*_{p,\mu}(f(0,0), f(1,1))} + \frac{(1-p)\alpha^*_{p,\mu}(f(0,0), f(1,1))}{p\mu + (1-p)\alpha^*_{p,\mu}(f(0,0), f(1,1))}\mu \\ \geq \frac{(1-p)\alpha^*_{p,\mu}(f(0,0), f(1,1))}{p\mu + (1-p)\alpha^*_{p,\mu}(f(0,0), f(1,1))}(1-\mu), \end{aligned} \tag{33}$$

which always holds. On the other hand, he optimally chooses action 0 upon receiving report 1 if his payoff from matching his action with the report is at most that from mismatching it:

$$\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\alpha^*_{p,\mu}(f(0,0),f(1,1)))} \\ + \frac{(1-p)(1-\alpha^*_{p,\mu}(f(0,0),f(1,1)))}{p(1-\mu)+(1-p)(1-\alpha^*_{p,\mu}(f(0,0),f(1,1)))}(1-\mu) \\ \leq \frac{(1-p)(1-\alpha^*_{p,\mu}(f(0,0),f(1,1)))}{p(1-\mu)+(1-p)(1-\alpha^*_{p,\mu}(f(0,0),f(1,1)))}\mu. \tag{34}$$

and matches his action with report 1 otherwise. This inequality simplifies to (7).

C.6 Proof of Lemma 7

For all $\mu \in (\frac{1}{2}, 1)$, $\kappa_{p,\mu} < 1$ by definition of (6). This lemma then follows by part 3 of Lemma 5, as $\alpha^*_{p,\mu}$, given in (32), is equal to one and so (7) fails for μ sufficiently close to one.

C.7 Proof of Lemma 8

By using (32) with $f(0,0) = f(1,1) = 1$, direct calculations show that (7) simplifies to $\mu \in C(p)$.

D Proof of existence in Proposition 1

I prove this claim by means of a general function $f : M \times S \to [0,1]$ with $f(0,1) = f(1,0) = 0$, and $f(0,0), f(1,1)$ satisfying (4). The special case with $f(0,0) = f(1,1) = 1$ proves the existence claim in Proposition 1. The following strategy profile constitutes a reputation equilibrium in which period 1 is informative. In period 1, $\sigma^I_1(0|0) = \sigma^I(1|1) = 1$ and $\sigma^U_1(0) = \alpha^*_{p,\mu}(f(0,0), f(1,1))$, where $\alpha^*_{p,\mu}$ is given in (32). By (4), $\alpha^*_{p,\mu} \in (0,1]$. The principal matches his action with the expert's report if $\alpha^*_{p,\mu} > \kappa_{p,\mu}$, where $\kappa_{p,\mu}$ is given in (6), and takes action 0 otherwise. In period 2, messages are drawn from S and

$$\sigma_2^I(0|m_1, s_1, s_2 = 0) = \begin{cases} 1, & \text{if } m_1 = s_1, \\ 1/2, & \text{if } m_1 \neq s_1. \end{cases}$$

$$\sigma_2^I(1|m_1, s_1, s_2 = 1) = \begin{cases} f(m_1, s_1), & \text{if } m_1 = s_1, \\ 1/2, & \text{if } m_1 \neq s_1. \end{cases}$$

$$\sigma_2^U(0|m_1, s_1) = \begin{cases} 1, & \text{if } m_1 = s_1, \\ 1/2, & \text{if } m_1 \neq s_1. \end{cases}$$

and the principal matches his action with the expert's report if $m_1 = s_1$ and chooses action 0 if $m_1 \neq s_1$. By construction, if history h_2 induces reputation p_2, then

$$w_2(h_2) = \begin{cases} \max\left[p_2\left(\mu + (1-\mu)f(m_1, s_1)\right) + (1-p_2)\mu - \mu, 0\right], & \text{if } m_1 = s_1, \\ \max[p_2 \frac{1}{2} + (1-p_2)\frac{1}{2} - \mu, 0], & \text{if } m_1 \neq s_1. \end{cases}$$

$$= \begin{cases} f(m_1, s_1)\bar{w}(p_2), & \text{if } m_1 = s_1, \\ 0, & \text{if } m_1 \neq s_1. \end{cases}$$

This wage follows because the principal's best reply to an expert report is to pick one of two options: matching his action with the report, in which case the wage is the action utility net of his reservation utility μ, or choosing action 0 irrespective of the report, in which case the wage is zero.

This profile constitutes an equilibrium. Following each period-2 history, neither expert type has a profitable deviation and the principal plays his myopic best reply. In period 1, the informed type has no profitable deviation because $w_2(m_1, s_1) = 0$ if $m_1 \neq s_1$. The uninformed type's best reply problem is given by (5) evaluated at $f(0,0) = f(0,0)$ and $f(1,1) = f(1,1)$, and by Lemma 5, her best reply is precisely $\alpha^*_{p,\mu}(f(0,0), f(1,1))$. By Lemma 6, the period-1 principal's strategy is her myopic best reply.

It is clear that in this equilibrium, period 1 is informative. This equilibrium also satisfies Property 1. In period 2 on path with reputation $p_2 > 0$, the period-2 wage is $p_2(1-\mu) > 0$ and is strictly increasing in p_2.

E Proof of Corollary 1

I first prove that if $\mu \geq 1/(1+p)$, then there is no reputation equilibrium with an informative period 1 and efficient period-2 play irrespective of past play. Suppose, towards a contradiction, that such an equilibrium exists. Define $\tilde{p} := \varphi(1,0)$. The arguments in Lemma 2 extend to this setting, and so it is without loss to assume that in period 1, both types' messages are drawn from S and the informed type reports the true state. Since the solution concept puts no restriction on off-path beliefs, suppose further that if history $h_2 = (m_1, s_1) = (1,0)$ occurs off path, then $\tilde{p} > p - (1-\mu)/\mu \geq 0$, where the second inequality follows from the assumption that $\mu \geq 1/(1+p)$. By (3), $\mu\bar{w}(p) < (1-\mu)\bar{w}(1) + \mu\bar{w}(\tilde{p})$. This ensures that if the period-2 principal conjectures the uninformed type to only report 0 in the first period, then the uninformed type has a profitable deviation to report 1 in that period. In equilibrium then, $\sigma_1^U(0) < 1$. By Claim 9, $\sigma_1^U(0) > 0$ and so $\sigma_1^U(0) \in (0,1)$ and the uninformed type must be indifferent between reporting either state:

$$\mu\bar{w}\left(\frac{p}{p+(1-p)\sigma_1^U(0)}\right) = (1-\mu)\bar{w}\left(\frac{p}{p+(1-p)(1-\sigma_1^U(0))}\right). \tag{35}$$

But the left side of (35) strictly exceeds $p\mu$ and the right side of (35) is strictly smaller than $1-\mu$, contradicting $\mu \geq 1/(1+p)$.

To prove the converse, suppose that $\mu < 1/(1+p)$. In this case, $\mu\bar{w}(p) < (1-\mu)\bar{w}(1)$ so that with efficient period-2 play irrespective of past play, if the principals conjecture that the uninformed type reports 0 with probability one in period 1, then in period 1, irrespective of the off-path reputation associated with an incorrect report 1 in this period, the uninformed type's best reply is to report 1 for sure, contrary to the principals' conjecture. The arguments in Appendix D can be directly adopted to show that the following strategy profile constitutes a reputation equilibrium with an informative period 1 and efficient period-2 play irrespective of past play. Specifically, in period 1, the informed type reports the state s_1 truthfully and the uninformed type reports 0 with probability $\alpha^*_{p,\mu}(1,1)$ that solves (5), given in (32). Because $\mu < 1/(1+p)$, $\alpha^*_{p,\mu}(1,1) \in (0,1)$ and so there is no off-path incorrect period-1 report. The principal matches his action with the expert's report if $\alpha^*_{p,\mu} \geq \kappa_{p,\mu}$, where $\kappa_{p,\mu}$ is given in (6), and chooses action 0 irrespective of the expert's report otherwise. In period 2, irrespective of the public history h_2, the informed type reports the state s_2 truthfully, the uninformed type reports 0 with probability one, and the principal matches his action with the expert's report. This completes the proof.

F Proofs for Section 6

F.1 Proof of Lemma 9

The proof in Appendix D applies.

F.2 Proof of Lemma 10

The following claim is useful.

Claim 10. *A reputation equilibrium is socially optimal if and only if this equilibrium maximizes the expert's ex ante lifetime wages*

$$\mathbf{E}[w_1 + w_2] = \mathbf{E}[u(a_1, s_1) + u(a_2, s_2)] - 2\mu \tag{36}$$

among all reputation equilibria.

Proof of Claim 10. This follows as (36) is a sum of (8) and a constant. ■

Consider a strategy profile $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$ given which $f(m_1, s_1) > 0$ if $m_1 = s_1$, $f(m_1, s_1) = 0$ if $m_1 \neq s_1$, and (4) holds. In period 1, $\sigma_1^I(s_1|s_1) = 1$ and $\sigma_1^U(0) = \alpha_{p,\mu}^*(f(0,0), f(1,1))$ where $\alpha_{p,\mu}^*(f(0,0), f(1,1))$ solves (5). This profile constitutes a reputation equilibrium, as Appendix D shows. Call this equilibrium a $(f(0,0), f(1,1))$-equilibrium. Note that $(f(0,0), f(1,1)) = (1, \mu p/(1-\mu))$ satisfies (4) and in a $(1, \mu p/(1-\mu))$-equlibrium, (36) is equal to

$$\bar{w}(p) + \mu\bar{w}(p) + (1-\mu)\frac{\mu p}{1-\mu}\bar{w}(1) = (1 + \mu - 2\mu^2)p. \tag{37}$$

Note also that in any socially optimal reputation equilibrium, the informed type's period-1 strategy is influential and thus also informative. If not, (41) is

$$\mathbf{E}[w_1] + \mathbf{E}[w_2(h_2)] = 0 + \mathbf{E}[w_2(h_2)] \leq \bar{w}(p),$$

where the belief p on the right side of the inequality follows from the martingale property of posterior beliefs. But $\bar{w}(p)$ is smaller than (37), contradicting social optimality in view of Claim 10.

Finally, consider a socially optimal reputation equilibrium with strategy profile σ and construct another profile $\tilde{\sigma}$ as in the proof of Lemma 2. By construction of $\tilde{\sigma}$, period 1 is informative and induces the same action distribution, and $\mathbf{E}[w_2(h_2)] = \tilde{\mathbf{E}}[\tilde{w}_2(h_2)]$, where $\tilde{\mathbf{E}}$ is the counterpart of $\mathbf{E}$ under $\tilde{\sigma}$. It remains to show that $w_1 = \tilde{w}_1$. Let $M(s)$ denote the set of messages inducing action s in period 1 under σ. By construction of $\tilde{\sigma}$, each period-1 message s induces action s under $\tilde{\sigma}$. By (1),

$$\begin{aligned} w_1 = p &\left(\mu \sum_{m\in M(0)} \sigma_1^I(m|0) + (1-\mu) \sum_{m\in M(1)} \sigma_1^I(m|1)\right) \\ &+ (1-p)\left(\mu \sum_{m\in M(0)} \sigma_1^U(m) + (1-\mu) \sum_{m\in M(1)} \sigma_1^U(m)\right) - \mu \\ = p &\left(\mu\tilde{\sigma}_1^I(0|0) + (1-\mu)\tilde{\sigma}_1^I(1|1)\right) + (1-p)\left(\mu\tilde{\sigma}_1^U(0) + (1-\mu)\tilde{\sigma}_1^U(1)\right) - \mu = \tilde{w}_1, \end{aligned}$$

as desired. Finally, as period 1 is influential and so informative, Lemma 2 applies and so, it is without loss to assume that $\tau_0 = \tau_1 = 1$.

F.3 Proof of Proposition 2

I first prove that (9) holds. Define

$$u(p;\alpha) := \; p + (1-p)(\mu\alpha + (1-\mu)(1-\alpha)). \tag{38}$$

Define

$$w^1_{p,\mu}(\alpha) := \; \max[u(p;\alpha) - \mu, 0]. \tag{39}$$

as the period-1 wage. This wage structure follows for the same reason as in Appendix D. By Lemmas 3 and 10, in any socially optimal reputation equilibrium, the expert's lifetime payoff (36) can be written as

$$\begin{aligned} g(\alpha; f) &:= w^1_{p,\mu}(\alpha) + \mathbf{E}[f(m_1,s_1)\bar{w}(\varphi(m_1,s_1))] \\ &= w^1_{p,\mu}(\alpha) + (\mu f(0,0) + (1-\mu)f(1,1))\bar{w}(p), \end{aligned} \tag{40}$$

given $f(0,1) = f(1,0) = 0$. An upper bound on the expert's lifetime payoff (36) at the social optimum is thus characterized by the value of the program

$$\max_{\substack{\alpha, f(0,0), f(1,0), \\ f(1,1), f(0,1) \in [0,1]}} g(\alpha; f) \tag{$\mathcal{P}$}$$

$$\text{s.t.} \quad f(0,0)\bar{w}\left(\frac{p}{p+(1-p)\alpha}\right) \geq 0, \tag{IC$_0$}$$

$$f(1,1)\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha)}\right) \geq 0, \tag{IC$_1$}$$

$$\alpha \in \arg\max_{\alpha' \in [0,1]} \mu\alpha' f(0,0)\bar{w}\left(\frac{p}{p+(1-p)\alpha}\right) \tag{IC$_U$}$$

$$+ (1-\mu)(1-\alpha')f(1,1)\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha)}\right).$$

Here, (IC$_0$) and (IC$_1$) capture the informed type's incentive constraint given period-1 state 0 and 1; (IC$_U$) captures the uninformed type's incentive constraint. In ($\mathcal{P}$), (IC$_0$) and (IC$_1$) always hold, and it is without loss to choose $f(1,0) = f(0,1) = 0$.

The value of ($\mathcal{P}$) is an upper bound because it is *a priori* unclear whether the solutions $(\alpha, f(0,0), f(1,1))$ to this program can be attained in some reputation equilibrium. The construction in Appendix D nonetheless shows that they can be attained, and therefore that a socially optimal reputation equilibrium exists in which the expert's lifetime payoff (41) is precisely the value of ($\mathcal{P}$).

Claim 11. *At the optimum of* ($\mathcal{P}$), $f(0,0) = 1$.

Proof of Claim 11. Suppose towards a contradiction that $(\alpha, f(0,0), f(1,1))$ constitutes a solution to ($\mathcal{P}$) in which $f(0,0) < 1$. Consider another tuple $(\hat{\alpha}, 1, f(1,1))$, in which $\hat{\alpha}$ is chosen to satisfy (IC$_U$) and so $\hat{\alpha} > \alpha$. This new tuple strictly improves (40). Contradiction. ■

The objective of ($\mathcal{P}$) at the social optimum then simplifies to

$$w^1_{p,\mu}(\alpha) + \mu(p+(1-p)\alpha)\bar{w}\left(\frac{p}{p+(1-p)\alpha}\right)$$

$$+ (1-\mu)(p+(1-p)(1-\alpha))f(1,1)\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha)}\right).$$

$$= w^1_{p,\mu}(\alpha) + \mu\bar{w}(p) + (1-\mu)f(1,1)\bar{w}(p), \tag{41}$$

By Claim 9, $\alpha > 0$ at the optimum of ($\mathcal{P}$). (IC_U) then simplifies to (5) with $f(0,0) = 1$. If $\mu p \geq 1 - \mu$, then for every $f(1,1) \in [0,1]$, there is no $\alpha \in (0,1)$ that satisfies (IC_U), and so $\alpha = 1$ at the optimum. In turn, given $\alpha = 1$, $f(1,1) = 1$ since (36) is strictly increasing in $f(1,1)$ and setting $f(1,1) = 1$ does not violate (IC_U). Suppose instead $\mu p < 1 - \mu$. Then (IC_U) implies that either $f(1,1) < 1$ or $\alpha < 1$ at the optimum. In addition, at the optimum, $f(1,1)$ must satisfy:

$$\mu\alpha\bar{w}\left(\frac{p}{p+(1-p)\alpha}\right) = (1-\mu)(1-\alpha)f(1,1)\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha)}\right). \tag{42}$$

The reason is as follows. If $\alpha = 1$ at the optimum, then $f(1,1) < 1$ and (IC_U) implies that the left side is at least the right side in (42). Suppose towards a contradiction that the left side is strictly higher than the right side, then $f(1,1)$ can be increased without disrupting the feasibility of choosing $\alpha = 1$. This increase strictly improves the objective; a contradiction. On the other hand, if $\alpha \in (0,1)$, then (IC_U) immediately requires that (42) holds. Substituting $f(1,1)$ that satisfies (42) into (41), (41) becomes

$$w^1_{p,\mu}(\alpha) + \left[\mu + \mu\frac{p+(1-p)(1-\alpha)}{p+(1-p)\alpha}\right]p(1-\mu).$$

This objective is strictly convex in α. Thus, there are two candidates $(\alpha, f(1,1))$ that attains the optimum of ($\mathcal{P}$), given by

$$\left(1, \frac{\mu p}{1-\mu}\right), \tag{43}$$

$$\text{and} \quad \left(\mu\left(\frac{1+p}{1-p}\right) - \frac{p}{1-p}, 1\right). \tag{44}$$

To complete the proof of (9), it suffices to compare the expert's *ex ante* lifetime payoffs (36) under both candidates. Under solution (43), this lifetime payoff is

$$p(1-\mu) + \mu(1-\mu)p + \mu(1-\mu)p^2. \tag{45}$$

Under solution (44), this lifetime payoff is

$$\max\left[(1-2(1-\mu)\mu)(1+p) - \mu, 0\right] + \mu(1-\mu)p + (1-\mu)^2 p. \tag{46}$$

(45) exceeds (46) if and only if

$$\mu > \frac{1+p}{2+p}.$$

This proves (9). Conversely, a socially optimal reputation equilibrium exists if a reputation equilibrium exists in which f satisfies (9) and so attains the value of the program ($\mathcal{P}$). Appendix D ensures that this equilibrium exists.

F.4 Proof of Corollary 2

Influentiality at the social optimum is shown in the proof of Lemma 10.

G Proofs for Section 7

G.1 Proof of Proposition 3

Fix a reputation equilibrium satisfying the conditions stated in the corollary. By (40), social welfare (8) is given by

$$(\mu + \max[u(p; \alpha^*_{p,\mu}(1, f(1,1;\mu))) - \mu, 0]) + (\mu + (\mu + (1-\mu)f(1,1;\mu)))\bar{w}(p)),$$

where $\alpha^*_{p,\mu}$ is given in (32) and $u(p;\cdot)$ is defined in (38). Direct calculations show that this is strictly increasing in μ.

G.2 Proof of Corollary 3

This claim is used in the proof of Proposition 3 and follows from direct calculations.

H Renegotiation-proofness

In this appendix, I consider Property 2 below, adopted from Ely and Välimäki (2003, Assumption 1):

Property 2. *At any history h_2 inducing reputation one, $f(h_2) = 1$.*

This property could rule out socially optimal reputation equilibria:

Corollary 4. *If* $\mu \in (\min\left[\frac{1+p}{2+p}, \frac{1}{1+p}\right], \frac{1}{1+p})$*, no socially optimally reputation equilibrium satisfying Property 2 exists. Otherwise, the equilibrium in Proposition 2 is a socially optimal reputation equilibrium satisfying Property 2.*

Intuitively, over intermediate values of μ, the players desire to coordinate on choosing $f(1,1)$ to be sufficiently small so that the uninformed type reports 0 with probability arbitrarily close to one, but not one, so that Property 2 has no bite. There is then no reputation equilibrium that attains the maximum of (8).

Proof of Corollary 4. Let $(\alpha, f(1,1))$ be as defined in the proof of Proposition 2. As shown in that proof, if

$$\mu \in \left(\min\left[\frac{1+p}{2+p}, \frac{1}{1+p}\right], \frac{1}{1+p}\right), \tag{47}$$

then the social optimum calls for $(\alpha, f(1,1))$ to be equal to (43), and the resulting expert's equilibrium lifetime payoff (45) strictly exceeds (46) that is sustained by the other candidate (44). The solution (43) however violates Property 2.

Suppose that (p, μ) satisfies (47) and so $\alpha^*_{p,\mu}(1,1) < 1$. Let $\{\varepsilon^n\}_{n=0}^{\infty}$ denote a decreasing sequence converging to zero and define, for each n,

$$\gamma_1^n := \frac{\mu(p + \varepsilon^n(1-p) - p)}{(1-\mu)(1-\varepsilon^n(1-p))}. \tag{48}$$

with $\varepsilon^n > 0$ chosen to be sufficiently small so that $f(1,1) \in [0,1]$. By (32), $\alpha^*_{p,\mu}(1, \gamma_1^n) = 1 - \varepsilon^n$. The sequence $\{(1-\varepsilon^n, \gamma_1^n)\}_{n=0}^{\infty}$ converges to (43). For each n, the $(1-\varepsilon^n, \gamma_1^n)$-equilibrium (defined in the proof of Lemma 10) is a reputation equilibrium that satisfies Property 2. The expert's lifetime payoff is

$$\begin{aligned} &p + (1-p)(\mu(1-\varepsilon^n) + (1-\mu)\varepsilon^n) - \mu \\ &\quad + \mu \bar{w}\left(\frac{p}{p+(1-p)(1-\varepsilon^n)}\right) + (1-\mu)\gamma_1^n \bar{w}\left(\frac{p}{p+(1-p)\varepsilon^n}\right) \end{aligned} \tag{49}$$

This payoff tends to (45) as $n \to \infty$. Thus, for sufficiently large n, (49) is strictly higher than (46) given that (p, μ) satisfies (47). But (45) can only be sustained by picking $(\alpha, f(1,1))$ to be exactly equal to (43) that violates Property 2; thus, no socially optimal reputation equilibrium exists. ■

I Extensions

In this appendix, I elaborate on the extensions mentioned in the main text.

I.1 Optional hiring

Proposition 1 extends if a principal can choose to not hire the expert, in which case the expert does not send a message. Suppose that in period 1, the principal does not hire the expert if he anticipates to choose action 0 irrespective of the expert's message. In reputation equilibria, this happens if and only if $\alpha^*_{p,\mu} \leq \kappa_{p,\mu}$, as in the main analysis.

I.2 Richer set of states, state transitions, and beliefs

My analysis readily extends to more than two states (and actions) without affecting my insights. This is because each principal's best-reply problem is to decide whether to match his action with an expert report or to choose the action that public information deems optimal, but not any other action even if there are more than two states.

The arguments in the proof of Proposition 1 readily extend if μ is randomly drawn from some distribution in each period and this distribution can depend on the public history, so long as the value of this draw is publicly observable.

Finally, for the qualitative predictions in Proposition 1, it is important that the uninformed type and the principal's prior state beliefs agree on the same state as being more likely to be true, but not that they are identical. This is because in reputation equilibria, in each informative period, a principal matches his action with an expert report if and only if he perceives uninformed gambling as sufficiently unlikely relative his own state belief, as in Lemma 6. This latter event happens when the uninformed type's prior information has sufficiently high quality so that she is unlikely to gamble, akin to the condition $\mu > \bar{\mu}$ in Proposition 1, or when the principal's prior information has sufficiently low quality so that he values expert advice more than his own information, akin to the condition $\mu < \bar{\mu}$ in Proposition 1.

Supplementary Appendices (for online publication)

J Perturbations

In this appendix, I extend my results to settings in which some informational assumptions of my model are perturbed. Section J.1 considers an extension in which in each period, the informed type observes an imperfect private signal about the state, but not the state, and this signal is sufficiently precise and more precise than public information is. Section J.2 considers an extension in which the state is sufficiently likely, but not certain, to be publicly realized when each period ends.

J.1 Sufficiently precise informed type's signals

In this section, suppose that in each period t, the informed type does not see the state s_t but sees a noisy private signal $y_t \in S$ before sending a message. This signal y_t is drawn to be equal to the true state s_t with probability $z \in (\mu, 1)$ and be different from s_t otherwise. Here, z captures the informed type's signal quality and by assumption, this quality is always higher than that of public information about the state. In each period t, the informed type's history is $h_t^I = (h_t, y_t)$. The model is otherwise unchanged. As in the main text, I often omit the null history h_1 in the notations.

In this extension, at each history $h_1^I = y_1$, the informed type's strategy $\sigma_1^I(y_1)$ induces the following distribution over her period-2 reputations p_2 (given the principals' conjecture of equilibrium strategies):

$$z \circ \mathbf{P}[\theta = I | m_1, s_1 = y_1] + (1 - z) \circ \mathbf{P}[\theta = I | m_1, s_1 \neq y_1]. \tag{50}$$

This distribution is non-degenerate if the informed type's period-1 strategy is informative, contrary to the case in the main analysis in which $z = 1$.

I focus on equilibria satisfying Property 1′ below that minimally relaxes Property 1. (Any equilibrium that satisfies Property 1′ must satisfy Property 1.) I call equilibria satisfying Properties 1′ weak reputation equilibria.

Property 1′. *The following holds.*

1. *Gven any two period-2 public histories h_2 and $\hat{h}_2$ on path inducing reputations p_2 and $\hat{p}_2$, provided that $w_2(h_2) > 0$ and $w_2(\hat{h}_2) > 0$, $w_2(h_2) \geq w_2(\hat{h}_2)$ if and only if $p_2 \geq \hat{p}_2$.*

2. *There exists $z^* \in (0,1)$ such that if $z \geq z^*$ and the informed type's period-1 strategy σ_1^I is informative, then for every y_1, among all distributions (50) over reputations that her strategy $\sigma_1^I(y_1)$ can induce, there is a unique distribution that maximizes her expected period-2 wage.*

Different from Property 1, part 1 of Property 1′ allows players to coordinate on not utilizing the value of an expert with a positive reputation, i.e., to coordinate on zero period-2 wage. This relaxation is useful in punishing incorrect reports and in turn maintaining the informed type's incentive to play an informative strategy in period 1.

Part 2 is a continuity condition. In my main model in which $z = 1$ in (50), the distribution (50) that the informed type's informative period-1 strategy at each s_1 induces is unique since her incentive to report the true state is strict in reputation equilibria. Part 2 ensures that when z falls short of one but is sufficiently close to one, the distribution (50) that the informed type's informative period-1 strategy at each y_1 induces is also unique, allowing Lemma 2 to extend:

Lemma 11. *Suppose that $z \geq z^*$. If a weak reputation equilibrium with an informative period 1 exists, then there exists a weak reputation equilibrium with an informative period 1 and with an identical period-1 action distribution, in which each expert type draws her period-1 message from S and the informed type truthfully reports her signal y_1 in period 1.*

The proof is in Appendix K.1. Given Lemma 11, I assume without loss for proving Proposition 1′ below that in any weak reputation equilibrium, each expert type draws her message from S and the informed type truthfully reports her signal in period 1.

In any weak reputation equilibrium, there must be some history h_2 on path at which $w_2(h_2) > 0$, for otherwise the informed type is indifferent among all messages in an informative period 1, violating part 2 of Property 1′; following this history, the informed type's strategy is influential. Following any other history h_2' at which $w_2(h_2') = 0$, the informed type's strategy is non-influential and so it is without loss to assume that this wage is supported by a babbling continuation.

In any weak reputation equilibrium, in line with part 1 of Property 1′, I say that period-2 play is weakly efficient on path if for each history h_2 on path satisfying $w_2(h_2) > 0$, the informed type induces this principal to take an action that matches her signal y_2 and the uninformed type induces him to take action 0.

Proposition 1′ below extends Proposition 1.

Proposition 1′. *There exists $\underline{z} \in [z^*, 1)$ such that for every $z \geq \underline{z}$, the following holds. In each weak reputation equilibrium with an informative period 1:*

1. *There exists $\bar{\mu}_z \equiv \bar{\mu}_z(p) \in (\frac{1}{2}, 1]$ such that for every $\mu \geq \bar{\mu}_z$, the informed type's (informative) strategy is influential.*
2. *If, in addition, period-2 play is weakly efficient on path, then the informed type's (informative) period-1 strategy σ_1^I is non-influential if and only if $\mu \in C_z(p)$ for some correspondence $C_z : (0, 1) \rightrightarrows [\frac{1}{2}, 1)$ satisfying:*
   - *(a) If $C_z(p)$ is nonempty, then $C_z(p) = [\underline{\mu}_z, \bar{\mu}_z]$ for some $\underline{\mu}_z \equiv \underline{\mu}_z(p)$ satisfying $\frac{1}{2} < \underline{\mu}_z \leq \bar{\mu}_z < 1$, where $\bar{\mu}_z$ is given in part 1.*
   - *(b) There exists $\bar{p}_z \in (0, 1)$ such that $C_z(p)$ is nonempty if and only if $p \leq \bar{p}_z$. For any p, p' satisfying $0 < p < p' \leq \bar{p}_z$, $C_z(p') \subsetneq C_z(p)$.*

A weak reputation equilibrium with an informative period 1 and weakly efficient period-2 play on path exists.

The proof of this proposition is in Appendix K.2.

J.2 Sufficiently likely public state realizations

In this section, suppose that at the end of each period, the state is publicly realized with some probability $r \in (0, 1)$ and remains hidden with probability $1 - r$. Here, the period-2 public history h_2 is (m_1, s_1) if the state s_1 is publicly realized and is m_1 otherwise. The model is otherwise identical to that in Section 2.

In this extension, the distribution over period-2 reputations p_2 that the informed type's period-1 strategy $\sigma_1^I(s_1)$ at each history $h_1^I = s_1$ induces (given the principals' conjecture of the equilibrium strategies) is

$$r \circ \mathbf{P}[\theta = I | m_1, s_1] + (1 - r) \circ \mathbf{P}[\theta = I | m_1]. \tag{51}$$

I focus on equilibria satisfying a modification of Property 1′ that suits this extension and is called Property 1″ below. Abusing terminology, I also refer to equilibria satisfying Property 1″ as weak reputation equilibria.

Here, contrary to Property 1′, there is no need to relax Property 1 by allowing the players to coordinate on not utilizing the value of an expert with a positive reputation.

Property 1″. *The following holds.*

1. *Given any two period-2 public histories h_2 and $\hat{h}_2$ on path inducing reputations p_2 and $\hat{p}_2$, $w_2(h_2) \geq w_2(\hat{h}_2)$ if and only if $p_2 \geq \hat{p}_2$. In addition, if $p_2 > 0$, then $w_2(h_2) > 0$.*
2. *There exists $r^* \in (0,1)$ such that if $r \geq r^*$, then the informed type's period-1 strategy σ_1^I is informative, then for every s_1, among all distributions (50) over reputations that her strategy $\sigma_1^I(s_1)$ can induce, there is a unique distribution that maximizes her expected period-2 wage.*

As in Appendix J.1, the following lemma holds.

Lemma 12. *Suppose that $r \geq r^*$. If a weak reputation equilibrium with an informative period 1 exists, then there exists a weak reputation equilibrium with an informative period 1 and with an identical period-1 action distribution, in which each expert type draws her period-1 message from S and the informed type truthfully reports the state in period* 1*.*

I omit its proof, as the arguments are identical to those in proving Lemma 11. In view of Lemma 12, in proving Proposition 1″ below, I assume without loss that in any weak reputation equilibrium, each expert type draws her message from S and the informed type truthfully reports the state in period 1, and players coordinate on babbling upon an incorrect period-1 report.

As in the main text, in any weak reputation equilibrium, at any period-2 public history on path conditional on the expert being informed, this expert's strategy must be influential by part 1 of Property 1″.

Proposition 1″ below extends Proposition 1.

Proposition 1″. *There exists $\underline{r} \in [r^*, 1)$ such that for every $r \geq \underline{r}$, the following holds. In each weak reputation equilibrium with an informative period 1:*

1. *There exists $\bar{\mu}_r \equiv \bar{\mu}_r(p) \in (\frac{1}{2}, 1]$ such that for every $\mu \geq \bar{\mu}_r$, the informed type's (informative) strategy is influential.*
2. *If, in addition, period-2 play is efficient on path, then the informed type's period-1 strategy σ_1^I is non-influential if and only if $\mu \in C_r(p)$ for some correspondence $C_r : (0,1) \rightrightarrows [\frac{1}{2}, 1)$ satisfying:*
   (a) *If $C_r(p)$ is nonempty, then $C_r(p) = [\underline{\mu}_r, \bar{\mu}_r]$ for some $\underline{\mu}_r \equiv \underline{\mu}_r(p)$ satisfying $\frac{1}{2} < \underline{\mu}_r \leq \bar{\mu}_r < 1$, where $\bar{\mu}_r$ is given in part 1.*

(b) There exists $\bar{p}_r \in (0,1)$ such that $C_r(p)$ is nonempty if and only if $p \leq \bar{p}_r$. For any p, p' satisfying $0 < p < p' \leq \bar{p}_r$, $C_r(p') \subsetneq C_r(p)$.

A weak reputation equilibrium with an informative period 1 and with efficient period-2 play exists.

The proof of this proposition is in Appendix K.3.

K Proofs for Appendix J

K.1 Proof of Lemma 11

Fix $z \geq z^*$. Fix a weak reputation equilibrium with an informative period 1. Let $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$ denote the profile of strategies in this equilibrium. Let M denote the set of equilibrium messages. Relabeling the messages if necessary, assume without loss that $S \subsetneq M$. Fix $s_1 = s$ and $y_1 = y$. Suppose that there are two messages $m, m' \in M$ given which $\sigma_1^I(m|y) > 0$ and $\sigma_1^I(m'|y) > 0$. The informed type must be indifferent between sending m and m' at history $h_1^I = y$. By part 2 of Property 1′,

$$\varphi(m, s) = \varphi(m', s) \quad \text{and} \quad \varphi(m, \neg s) = \varphi(m', \neg s).$$

By the same arguments as given in Lemma 2, there is another weak reputation equilibrium with an informative period 1 and with an identical period-1 action distribution, in which each expert type draws her period-1 message from S. Next, fix such a reputation equilibrium and, abusing notations, let $\sigma := (\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=1,2}$ denote the profile of strategies in this equilibrium. Define $\tau_s := \sigma_1^I(y|y)$ and $\alpha := \sigma_1^U(0)$.

Claim 12. *It holds that*

$$\tau_0 \neq 1 - \tau_1, \tag{52}$$

$$\text{and} \qquad z\tau_0 + (1-z)(1-\tau_1) \neq \alpha \quad \text{or} \quad z\tau_1 + (1-z)(1-\tau_0) \neq 1 - \alpha. \tag{53}$$

Proof of Claim 12. By Definition 1,

$$\mu \neq \mathbf{P}[s_1 = 0 | m_1 = 0]$$
$$= \frac{\mu(p(z\tau_0 + (1-z)\tau_1) + (1-p)\alpha)}{\mu(p(z\tau_0 + (1-z)\tau_1) + (1-p)\alpha) + (1-\mu)(p(z(1-\tau_1) + (1-z)\tau_0) + (1-p)\alpha)},$$

or

$$\mu \neq \mathbf{P}[s_1 = 0 | m_1 = 1]$$
$$= \frac{\mu(p(z(1-\tau_0) + (1-z)\tau_1) + (1-p)(1-\alpha))}{\mu(p(z(1-\tau_0) + (1-z)\tau_1) + (1-p)(1-\alpha)) + (1-\mu)(p(z\tau_1 + (1-z)(1-\tau_0)) + (1-p)(1-\alpha))}.$$

Either expression requires (52) and (53). ■

Claim 13. *Either* $\tau_0 = \tau_1 = 1$ *or* $\tau_0 = \tau_1 = 0$.

Proof of Claim 8. I first show that $\tau_0, \tau_1 \in \{0, 1\}$. Suppose, towards a contradiction, that $\tau_0 \in (0, 1)$, then the informed type must be indifferent between reporting 0 or 1 given signal 0. This and part 2 of Property 1 imply:

$$\varphi(0,0) = \varphi(1,0) \quad \text{and} \quad \varphi(0,1) = \varphi(1,1). \tag{54}$$

These two equations are plainly

$$\frac{p(z\tau_0 + (1-z)(1-\tau_1))}{p(z\tau_0 + (1-z)(1-\tau_1)) + (1-p)\alpha} = \frac{p(z(1-\tau_0) + (1-z)\tau_1)}{p(z(1-\tau_0) + (1-z)\tau_1) + (1-p)(1-\alpha)},$$
$$\frac{p(z(1-\tau_1) + (1-z)\tau_0)}{p(z(1-\tau_1) + (1-z)\tau_0) + (1-p)\alpha} = \frac{p(z\tau_1 + (1-z)(1-\tau_0))}{p(z\tau_1 + (1-z)(1-\tau_0)) + (1-p)(1-\alpha)},$$

which further simplify to

$$z\tau_0 + (1-z)(1-\tau_1) = \alpha, \quad \text{and} \quad z\tau_1 + (1-z)(1-\tau_0) = 1-\alpha,$$

contradicting (53). Thus, $\tau_0 \in \{0, 1\}$ and analogously, $\tau_1 \in \{0, 1\}$. Finally, by (52), Claim 13 follows. ■

The remainder of the proof is as in Lemma 2.

K.2 Proof of Proposition 1′

Fix z^* in Property 1′. Fix $\underline{z} := \max\{z^*, \underline{z}', \underline{z}'', \underline{z}'''\}$, where $\underline{z}', \underline{z}'', \underline{z}''' \in (0, 1)$ are chosen below. Fix $z \geq \underline{z}$. The analog of (3) in this extension, with an abuse of the notation $\bar{w}$, is

$$\bar{w}(p_2) := \max(p_2 z + (1-p_2)\mu - \mu, 0) = p_2(z-\mu) > 0. \tag{55}$$

K.2.1 Parts 1 and 2

Fix a weak reputation equilibrium. By part 1 of Property 1′, the period-2 wage following public history (m_1, s_1) inducing reputation p_2 can be written as $\gamma_{s_1}\bar{w}(p_2)$ if $m_1 = s_1$ and as $\phi_{s_1}\bar{w}(p_2)$ if $m_1 \neq s_1$, for some $\gamma_{s_1}, \phi_{s_1} \in [0,1]$.

Define $\alpha := \sigma_1^U(0)$. In equilibrium, α solves

$$\begin{aligned}\alpha \in \arg\max_{\tilde{\alpha}\in[0,1]} \tilde{\alpha}\Bigg[&\mu\gamma_0\bar{w}\left(\frac{pz}{pz+(1-p)\alpha}\right)\\ &+(1-\mu)\phi_0\bar{w}\left(\frac{p(1-z)}{p(1-z)+(1-p)\alpha}\right)\Bigg]\\ &+(1-\tilde{\alpha})\Bigg[\mu\phi_1\bar{w}\left(\frac{p(1-z)}{p(1-z)+(1-p)(1-\alpha)}\right)\\ &+(1-\mu)\gamma_1\bar{w}\left(\frac{pz}{pz+(1-p)(1-\alpha)}\right)\Bigg].\end{aligned} \tag{56}$$

The period-1 principal chooses action 0 independently of the expert's report if

$$pz + (1-p)(\alpha\mu + (1-\alpha)(1-\mu)) \leq \mu, \tag{57}$$

and he matches his action with the expert's report otherwise. Rearranging (57) yields:

$$\alpha \leq \kappa_{p,\mu,z} := \frac{\mu(2-p)+p(1-z)-1}{(2\mu-1)(1-p)}. \tag{58}$$

Let $\{z_n\}_{n=0}^{\infty}$ be an increasing sequence converging to one, with $z_n \in (0,1)$ for each n. Let α_n denote the uninformed type's strategy that solves (56) with $z = z_n$. Note that $\{\alpha_n\}_{n=0}^{\infty}$ converges uniformly to $\alpha^*_{p,\mu}$ given in (5) and $\{\kappa_{p,\mu,z_n}\}_{n=0}^{\infty}$ converges uniformly to $\kappa_{p,\mu}$ given in (6). Then, there exists $\underline{z}' \in (0,1)$ such that for every $z \geq \underline{z}'$, parts 1 and 2 follow from the same arguments as in the proof of Lemma 6.

K.2.2 Existence

The following strategy profile constitutes a weak reputation equilibrium in which period 1 is informative and period-2 play is weakly efficient. In period 1, the informed type reports her signal truthfully; the uninformed type reports 0 with probability

$$\alpha^*_{p,\mu,z} := \min\left[1, \mu + \frac{(2\mu-1)pz}{1-p}\right]. \tag{59}$$

Note that $\alpha^*_{p,\mu,1} = \alpha^*_{p,\mu}$ given in (32). The principal matches his action with the expert's report if and only if $\alpha^*_{p,\mu,z} > \kappa_{p,\mu,z}$, where $\kappa_{p,\mu,z}$ is given in (58), and chooses action 0 otherwise. In period 2, following a correct period-1 report, the informed type reports her signal truthfully, the uninformed type reports 0, and the principal matches his action with the expert's report; following an incorrect period-1 report, both types report the two states with equal probabilities and the principal chooses action 0. By construction,

$$w_2(p_2; h_2) = \begin{cases} \bar{w}(p_2), & \text{if } m_1 = s_1, \\ 0, & \text{otherwise.} \end{cases}$$

I verify that for z sufficiently close to one, this profile constitutes an equilibrium. In period 2, neither expert type has a profitable deviation and the principal plays his myopic best reply. In period 1, given signal 0, the informed type's incentive constraint from reporting correctly is

$$z\bar{w}\left(\frac{pz}{pz + (1-p)\alpha^*_{p,\mu,z}}\right) \geq (1-z)\bar{w}\left(\frac{pz}{pz + (1-p)(1-\alpha^*_{p,\mu,z})}\right). \tag{60}$$

The expressions on both sides are continuous in z. At $z = 1$, the left side of (60) is bounded uniformly away from and above zero:

$$(1-\mu)\frac{p}{p + (1-p)\alpha^*_{p,\mu,1}} \geq (1-\mu)p > 0. \tag{61}$$

Also, at $z = 1$, the right side of (60) is equal to zero. By (61) and by continuity of z, there exists $\underline{z}''$ such that for every $z \geq \underline{z}''$, (60) holds strictly. For the same reason, there exists $\underline{z}'''$ such that for every $z \geq \underline{z}'''$ (60), the informed type's incentive constraint to report 1 given signal 1, namely

$$z\bar{w}\left(\frac{pz}{pz + (1-p)(1-\alpha^*_{p,\mu,z})}\right) \geq (1-z)\bar{w}\left(\frac{pz}{pz + (1-p)\alpha^*_{p,\mu,z}}\right). \tag{62}$$

also holds strictly. The uninformed type's strategy (59) is, by construction, her best reply that solves (56) with $f(0,0) = f(1,1) = 1$ and $\phi_0 = \phi_1 = 0$. Finally, by (58), the period-1 principal's strategy is her myopic best reply.

In this equilibrium, period 1 is informative. This equilibrium satisfies part 1 of Property 1′ because the expert's period-2 wage upon a correct period-1 report with reputation p_2 is $\bar{w}(p_2)$ by construction. This wage is strictly increasing in p_2 and is positive whenever $p_2 > 0$. On the other hand, her period-2 wage is zero upon an incorrect report despite having a positive reputation. Finally, this equilibrium satisfies part 2 of Property 1′ because the informed type's incentive constraint to report truthfully given each signal holds strictly in period 1.

K.3 Proof of Proposition 1″

Fix r^* in Property 1″. Fix $\underline{r} := \max\{r^*, \underline{r}', \underline{r}'', \underline{r}'''\}$, where $\underline{r}', \underline{r}'', \underline{r}''' \in (0, 1)$ are chosen below. Fix $r \geq \underline{r}$.

K.3.1 Parts 1 and 2

Fix a weak reputation equilibrium. By Part 1 of Property 1″, the period-2 wage given public history (m_1, s_1) or m_1 inducing reputation p_2 can be written as $\gamma_{m_1}\bar{w}(p_2)$ upon a correct report m_1 and $\phi_{m_1}\bar{w}(p_2)$ given a report m_1 without a state realization, for some $\gamma_{m_1}, \phi_{m_1} \in [0, 1]$, and zero otherwise.

Define $\alpha := \sigma_1^U(0)$. In equilibrium, because an incorrect period-1 reports leads to zero period-2 wages, α solves

$$\begin{aligned}\alpha \in \arg\max_{\tilde{\alpha}\in[0,1]} \tilde{\alpha}\left[r\mu f(0,0)\bar{w}\left(\frac{p}{p+(1-p)\alpha}\right) + (1-r)\phi_0\bar{w}\left(\frac{p\mu}{p\mu+(1-p)\alpha}\right)\right] \\ + (1-\tilde{\alpha})\left[r(1-\mu)f(1,1)\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha)}\right)\right. \\ \left. + (1-r)\phi_1\bar{w}\left(\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\alpha)}\right)\right].\end{aligned} \tag{63}$$

The period-1 principal takes action 0 independently of the expert's report if

$$p + (1-p)(\alpha\mu + (1-\alpha)(1-\mu)) \leq \mu, \tag{64}$$

and matches his action with the expert's report otherwise. Rearranging (64) yields $\alpha \leq \kappa_{p,\mu}$, where $\kappa_{p,\mu}$ is given in (6).

Let $\{r_n\}_{n=0}^{\infty}$ be an increasing sequence converging to one, with $r_n \in (0, 1)$ for each n. Let α_n denote the uninformed type's strategy that solves (63) with $r = r_n$. By construction, $\{\alpha_n\}_{n=0}^{\infty}$ converges uniformly to $\alpha^*_{p,\mu}$ given in (32). Then, there is $\underline{r}' \in (0, 1)$ such that for every $r \geq \underline{r}'$, parts 1 and 2 follow from the same arguments as in the proof of Lemma 6.

K.3.2 Existence

The following strategy profile constitutes a reputation equilibrium in which period 1 is informative. In period 1, the informed type reports the state truthfully; the uninformed type reports 0 with probability $\alpha^*_{p,\mu,r}$, defined as the unique solution that solves (63). Note that this solution is continuous in (p, μ, r), and $\alpha^*_{p,\mu,1} = \alpha^*_{p,\mu}$ given in (32). The principal matches his action with the expert's report if and only if $\alpha^*_{p,\mu,r} > \kappa_{p,\mu}$. In period 2, following a correct period-1 report or following no state realization in period 1, the informed type reports the state truthfully, the uninformed type reports 0, and the principal matches his action with the expert's report; following an incorrect period-1 report, both types report the two states with equal probabilities and the principal chooses action 0. By construction,

$$w_2(p_2; h_2) = \begin{cases} \bar{w}(p_2), & \text{if } h_2 = m_1 \text{ or if } h_2 = (m_1, s_1) \text{ with } m_1 = s_1, \\ 0, & \text{otherwise.} \end{cases}$$

I verify that for r sufficiently close to one, this profile constitutes an equilibrium. In period 2, neither expert type has a profitable deviation and the principal plays his myopic best reply. In period 1, given state observation 0, the informed type's incentive constraint to report truthfully is

$$r\bar{w}\left(\frac{p}{p + (1-p)\alpha^*_{p,\mu,r}}\right) + (1-r)\bar{w}\left(\frac{p\mu}{p\mu + (1-p)\alpha^*_{p,\mu,r}}\right) \geq (1-r)\bar{w}\left(\frac{p(1-\mu)}{p(1-\mu) + (1-p)(1-\alpha^*_{p,\mu,r})}\right). \tag{65}$$

The expressions on both sides are continuous in r. At $r = 1$, the left side of (65) is bounded uniformly away from and above zero:

$$\bar{w}\left(\frac{p}{p + (1-p)\alpha^*_{p,\mu,1}}\right) \geq (1-\mu)p > 0. \tag{66}$$

Also, at $r = 1$, the right side of (65) is equal to zero. By (66) and by continuity of r, $\underline{r}''$ is chosen such that for every $r \geq \underline{r}''$, (65) holds strictly. For the same reason, $\underline{r}'''$ is chosen such that for every $r \geq \underline{r}'''$, the informed type's incentive constraint to truthfully report state 1, namely

$$r\bar{w}\left(\frac{p}{p+(1-p)(1-\alpha^*_{p,\mu,r})}\right)+(1-r)\bar{w}\left(\frac{p(1-\mu)}{p(1-\mu)+(1-p)(1-\alpha^*_{p,\mu,r})}\right) \geq (1-r)\bar{w}\left(\frac{p\mu}{p\mu+(1-p)\alpha^*_{p,\mu,r}}\right) \tag{67}$$

also holds strictly. The uninformed type's strategy (59) is, by construction, her best reply that solves (56) with $f(0,0) = f(1,1) = \phi_1 = \phi_1 = 1$. Finally, by (65), the period-1 principal's strategy is her myopic best reply.

It is clear that in this equilibrium, period 1 is informative. This equilibrium satisfies part 1 of Property 1″ because the expert's period-2 wage upon a correct period-1 report or no period-1 state realization, with reputation p_2 is $\bar{w}(p_2)$ by construction, which is strictly increasing in p_2 and is positive if $p_2 > 0$. Finally, this equilibrium satisfies part 2 of Property 1″ since the informed type's incentive constraint to report truthfully given each history holds strictly in period 1.

L Longer horizon

In this appendix, I illustrate with an example that extending my analysis to more than two periods does not affect my insights. The example features three periods; as will be evident, the arguments readily extend to more periods in the natural way. I then demonstrate that a complete extension of the crisis region C, namely the interval structure as characterized in parts 1 and 2 of Proposition 1, is nonetheless analytically difficult to obtain.

Suppose that there are three periods $t = 1, 2, 3$ instead of two. The definitions of histories and strategies extend in the natural way. Given an equilibrium, let $v^\theta_t(h_t)$ denote type-θ expert's (expected) continuation payoff following public history h_t in period t. Extend Property 1 as follows.

Property 3. *In each period $t = 2, 3$, for each expert type θ and given any two period-t public histories h_t and $\hat{h}_t$ on path inducing reputations p_t and $\hat{p}_t$, $v^\theta_t(h_t) \geq v^\theta_t(\hat{h}_t)$ if and only if $p_t \geq \hat{p}_t$. In addition, if $p_t > 0$, then $v^\theta_t(h_t) > 0$.*

Next, in this extension, an additional equilibrium property is required to ensure that the expert's messages on path only serve to convey information about the payoff-relevant fundamentals, namely the state and the expert's type, but not to coordinate players' selection of different future play. In any equilibrium, each period-2 public history h_2 induces a payoff-relevant history $\pi_2 \equiv \pi_2(h_2) := ((p_1, s_1), p_2)$, consisting of the past reputation, and state realization, and the current reputation. Multiple period-1 messages m_1 could arise on path and convey the same (or no) information, leading to different public histories h_2 that induce the same reputation p_2 in period 2. Thus, multiple public histories h_2 could arise on path, each featuring a different past message m_1 but inducing the same π_2. Property 4 requires that players coordinate on the same continuation play across these histories:

Property 4. *At any two period-2 public histories h_2 and $\hat{h}_2$ on path inducing the same payoff-relevant history π_2, the profile of continuation strategies $(\sigma_t^I, \sigma_t^U, \sigma_t^P)_{t=2,3}$ following history h_2 is identical to that following history $\hat{h}_2$.*

Property 4 ensures that each payoff-relevant history π_2 on path uniquely identifies the period-2 play. Thus, this π_2 is associated with a unique (*ex ante*) distribution $\rho_2(\pi_2)$ over the period-2 principal's actions; this distribution draws action a with probability

$$\sum_{m \in M} \mathbf{P}\left[m_2 = m | \pi_2\right] \mathbf{P}\left[a_2 = a | \pi_2, m_2 = m\right]. \tag{68}$$

In the remainder of this section, I say that an equilibrium satisfying Properties 3 and 4 is a reputation equilibrium. Given Properties 3 and 4, the statements and the proofs of Lemmas 1—4 extend, applying not only to an informative period 1 but also to an informative period 2 following each period-2 public history on path. In particular, Property 4 is key to extending Lemma 2: it ensures that the period-1 messages inducing the same period-2 reputation for each period-1 state induce the same continuation play; thus, the relabeling of these messages and the construction of continuation strategies described in that proof not only preserves the period-1 action distribution, but also preserves the period-2 action distributions on path. Hereafter, I assume without loss that in any reputation equilibrium and any informative period 1 and 2 with no incorrect past report on path, the informed type reports the true state and each type's continuation payoff is zero upon reporting incorrectly. Thus, as in the main text, the informed type's incentive constraint to report truthfully in each informative period 1 and 2 on path is immediately satisfied.

I construct a reputation equilibrium. Suppose that the informed type's strategy is informative at each history with no incorrect past report. Moreover, suppose that the strategy profile is such that in period 3 with no incorrect past report and with reputation p_3, the wage is equal to $\bar{w}(p_3)$, where $\bar{w}$ is given in (3); this wage can be supported in equilibrium because period 3 is the last period, as in Lemma 5.

In period 2 with no incorrect past report and with reputation p_2, the uninformed type faces an identical best reply problem as in (5) with $f(0,0) = f(1,1) = 1$. Thus, in this period, she reports 0 with probability $\alpha^*_{p_2,\mu}$ that solves (5) with $f(0,0) = f(1,1) = 1$; the expression of $\alpha^*_{p_2,\mu}$ is given in (32). The principal's best reply is as in Appendix D.

In turn, given a public history h_2 with no incorrect past report and with reputation p_2, the period-2 wage is given by $\max(u^*_{p_2,\mu} - \mu, 0)$, where $u^*_{\cdot,\cdot}$ is given in (11). The informed type's continuation payoff $v^I_2(h_2)$ can be written as

$$\begin{aligned}
W^I_2(p_2) :=\ & \max[u^*_{p_2,\mu} - \mu, 0] + \mu\bar{w}\left(\frac{p_2}{p_2 + (1-p_2)\alpha^*_{p_2,\mu}}\right) \\
& \qquad\qquad\qquad + (1-\mu)\bar{w}\left(\frac{p_2}{p_2 + (1-p_2)(1-\alpha^*_{p_2,\mu})}\right) \\
=\ & \begin{cases} (1-\mu)(1+p_2-\mu(1-p_2)), & \text{if } p_2 \geq \dfrac{1-\mu}{\mu}, \\ \max(0, \mu(2\mu(1+p_2) - 2p_2 - 3) + p_2 + 1) + \dfrac{2(1-\mu)p_2}{1+p_2}, & \text{if } p_2 < \dfrac{1-\mu}{\mu}, \end{cases}
\end{aligned}$$

and the uninformed type's continuation payoff $v^U_2(h_2)$ can be written as

$$\begin{aligned}
W^U_2(p_2) :=\ & \max[u^*_{p_2,\mu} - \mu, 0] + \mu\alpha^*_{p_2,\mu}\bar{w}\left(\frac{p_2}{p_2 + (1-p_2)\alpha^*_{p_2,\mu}}\right) \\
& \qquad\qquad + (1-\mu)(1-\alpha^*_{p_2,\mu})\bar{w}\left(\frac{p_2}{p_2 + (1-p_2)(1-\alpha^*_{p_2,\mu})}\right) \\
=\ & \begin{cases} p_2(1-\mu^2), & \text{if } p_2 \geq \dfrac{1-\mu}{\mu}, \\ \max(0, \mu(2\mu(1+p_2) - 2p_2 - 3) + p_2 + 1) + \dfrac{p_2(1-\mu)}{1+p_2}, & \text{if } p_2 < \dfrac{1-\mu}{\mu}. \end{cases}
\end{aligned}$$

Direct calculations verify that W^I_2 and W^U_2 are strictly increasing in p_2.

The uninformed type's period-1 strategy, i.e., her probability $\alpha^{**}_{p,\mu}$ to report 0, solves

$$\begin{aligned}
\alpha^{**}_{p,\mu} \in \operatorname*{argmax}_{\alpha\in[0,1]} \mu\alpha W^U_2 & \left(\frac{p}{p + (1-p)\alpha^{**}_{p,\mu}}\right) \\
& + (1-\mu)(1-\alpha)W^U_2\left(\frac{p}{p + (1-p)(1-\alpha^{**}_{p,\mu})}\right).
\end{aligned} \tag{69}$$

This characterization mirrors (5), with the period-2 wage replaced by the continuation

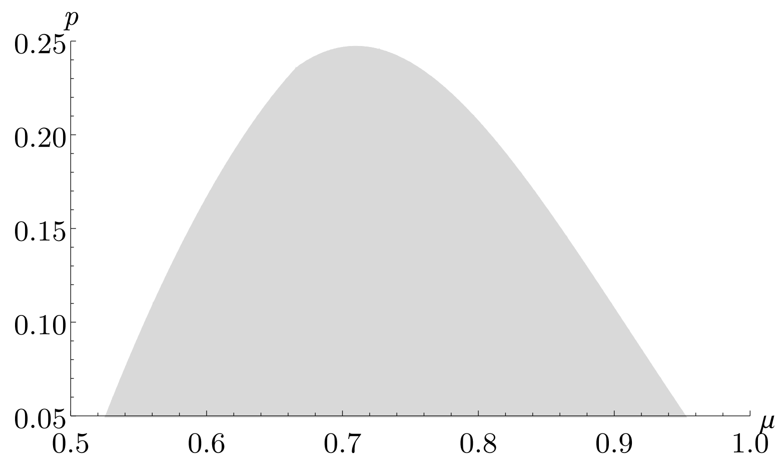


Figure 4: Crisis region $\hat{C}(p)$

payoff W_2^U. As in (5), (69) concerns the uninformed type's tradeoff between her future payoff upon a correct current report 0 and her future payoff upon a correct current report 1. In this period, the principal's best reply is again given as in Lemma 6. The strategy profile constructed above constitutes a reputation equilibrium, because Properties 3 and 4 are satisfied. It is clear from (69) that $\alpha^{**}_{p,\mu}$ is increasing in μ, so that better public information mitigates uninformed gambling as in the main text and the complementarity highlighted in the main text continues to hold.

Consider next the analog of parts 1 and 2 of Proposition 1 in this extension. Following a public history h_2 with no incorrect past report and with reputation p_2, the correspondence $C(p_2) = \{\mu \in [\frac{1}{2}, 1) : \alpha^*_{p_2,\mu} \leq \kappa_{p_2,\mu}\}$ is characterized as in Proposition 1. The counterpart of C in period 1 is given by

$$\hat{C}(p) := \{\mu \in [\tfrac{1}{2}, 1) : \alpha^{**}_{p,\mu} \leq \kappa_{p,\mu}\}. \tag{70}$$

From (69), it is clear that $\alpha^{**}_{p,\mu} = 1$ if μ is sufficiently close to one so that $\alpha^{**}_{p,\mu} > \kappa_{p,\mu}$. It is also clear that $\kappa_{p,\mu} \to -\infty$ as $\mu \to 1/2$ so that $\alpha^{**}_{p,\mu} > \kappa_{p,\mu}$ if μ is sufficiently close to 1/2. Thus, like $C(p)$, $\hat{C}(p)$ must exclude extreme values of μ and so the insight that high-quality public information preempts uninformed gambling continues to hold. It is however analytically difficult to conclude that $\hat{C}(p)$ is an interval whenever it is nonempty, because $\alpha^{**}_{p,\mu}$ lacks an analytically convenient structure. Numerical simulations are nonetheless straightforward, suggesting that $\hat{C}$ exhibits qualitatively identical properties as C does. Figure 4 illustrates.

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